Decentralized LQG Control

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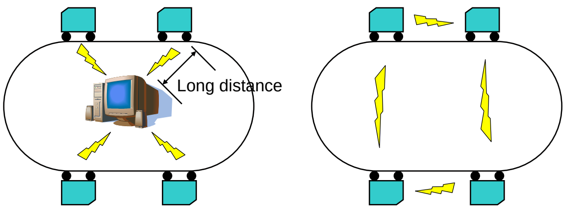
Outline

Motivation

Large-scale system (wide area, many control objects)



Centralized control

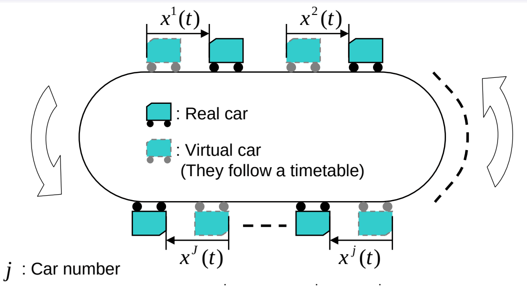
expensive, inefficient,
unreliable, inflexible

Decentralized control

economic, efficient,
reliable, flexible

Decentralized control is the better alternative for large-scale systems. [B1]

Considered System



j : Car number

$x^j(t)$: Position error

$$\ddot{x}^j(t) = -\alpha \dot{x}^j(t) + u^j(t)$$

Linear, time invariant system

α : Attenuation coefficient

System Equation

$$\ddot{x}^j(t) = -\alpha \dot{x}^j(t) + u^j(t)$$

$$x(t) = \begin{bmatrix} x^1(t) \\ \vdots \\ x^J(t) \\ \dot{x}^1(t) \\ \vdots \\ \dot{x}^J(t) \end{bmatrix} \text{ : State vector}$$

$$u(t) = \begin{bmatrix} u^1(t) \\ \vdots \\ u^J(t) \end{bmatrix} \text{ : Input vector}$$

$$\frac{d}{dt}x(t) = \underbrace{\begin{bmatrix} 0 & I_J \\ 0 & -\alpha I_J \end{bmatrix}}_{A_c} x(t) + \underbrace{\begin{bmatrix} 0 \\ I_J \end{bmatrix}}_{B_c} u(t)$$

$$\dot{x}(t) = A_c x(t) + B_c u(t)$$

Discretization

$$x_{k+1} = A x_k + B u_k$$

where

I_J : Identity matrix of $J \times J$

Centralized Control - LQR

Optimal regulator problem [B2], [B3]

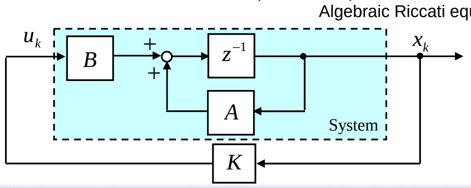
Quadratic cost function : $\phi = \sum_{k=0}^{N} (x_k^T Q x_k + u_k^T R u_k) \quad Q \geq 0, R > 0$

Optimal control : $u_k = K x_k$ Linear - Quadratic - Regulator (LQR)

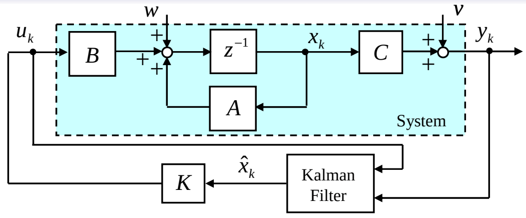
where $K = -(R + B^T P B)^{-1} B^T P A$

and $P = A^T P A + Q - A^T P B (R + B^T P B)^{-1} B^T P A \quad P \geq 0$

Algebraic Riccati equation



Centralized Control - consider noise



$$\begin{cases} x_{k+1} = Ax_k + Bu_k + w \\ y_k = Cx_k + v \end{cases}$$

$$E\{x_0\} = \bar{x}_0$$

$$E\{(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T\} = X_0$$

$\hat{x}_k$: Estimated state vector

Assume : w, v : white Gaussian noise

$$\begin{cases} E\{w\} = 0, E\{v\} = 0 \\ E\{ww^T\} = W \\ E\{vv^T\} = V \\ E\{wv^T\} = 0 \end{cases}$$

Measurement Equation

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + w \\ y_k = Cx_k + v \end{cases}$$

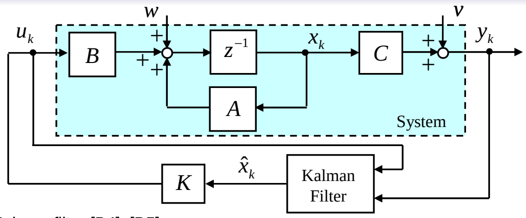
$$y^j(t) = \begin{bmatrix} x^j(t) \\ x^{j-1}(t) - x^j(t) \\ x^j(t) - x^{j+1}(t) \\ \dot{x}^j(t) \\ \dot{x}^{j-1}(t) - \dot{x}^j(t) \\ \dot{x}^j(t) - \dot{x}^{j+1}(t) \end{bmatrix} \quad y(t) = \begin{bmatrix} y^1(t) \\ \vdots \\ y^J(t) \end{bmatrix} \text{ : Output vector}$$

$$y(t) = C_c x(t) + v$$

Discretization ⇩

$$y_k = Cx_k + v$$

State Estimation and LQG



Kalman filter [B4], [B5]

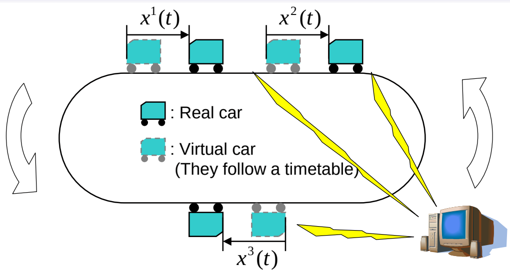
$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + L_k(y_k - C\hat{x}_k)$$

$$L_k = AS_kC^T(CS_kC^T + V)^{-1}$$

$$S_{k+1} = AS_kA^T - L_k(CS_kC^T + V)L_k^T + W \qquad \hat{x}_0 = \bar{x}_0, S_0 = X_0$$

$u_k = K\hat{x}_k$ Linear - Quadratic - Gaussian (LQG) K : LQR Gain

Centralized Control - Example

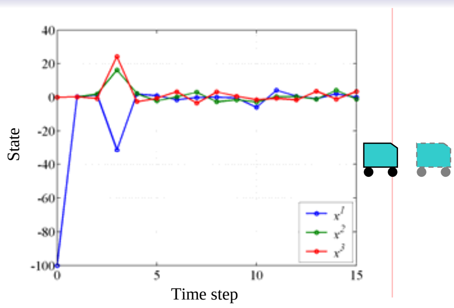


Desired: errors converge to 0 $\alpha = 1$

Initial state: $x_0 = [-100; 0; 0; 0; 0; 0]$ $\bar{x}_0 = 0, X_0 = I_J$

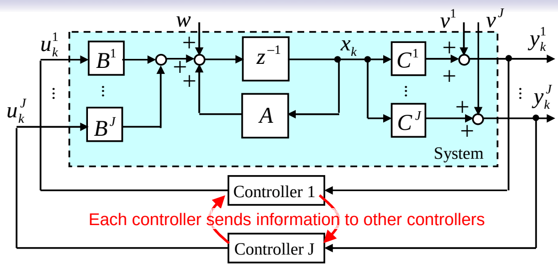
Weight matrix: $Q = 5I_6$, $R = 0.5I_3$ $W = 1, V = 1$

Simulation Result - LQG



Each error converge to 0 ⟶ Each car follow a timetable

Decentralized Control

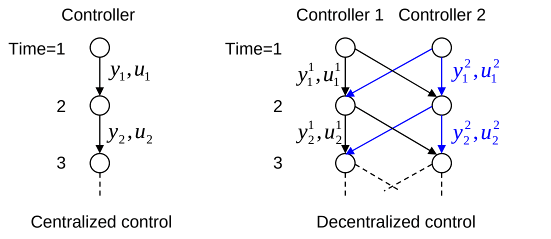


$$\begin{cases} x_{k+1} = Ax_k + \sum_{j=1}^{J} B^j u_k^j + w \\ y_k^j = C^j x_k + v^j \quad (j = 1, 2, \cdots, J) \end{cases}$$

$$\phi = \sum_{k=0}^{N} \left[x_k^T Q x_k + \sum_{j=1}^{J} u_k^{jT} R^j u_k^j \right]$$

$$Q \geq 0, R^j > 0$$

Infomation Structure



In this case, each controller can use all y_k^j, u_k^j

→ Each controller can be estimated x_{k+1} by Kalman Filter [B1]

Design of Controller

Put each matrix and each vector as follows

$$B_d = [B^1 \ \cdots \ B^J]$$

$$C_d = \begin{bmatrix} C^1 \\ \vdots \\ C^J \end{bmatrix} \quad u_k = \begin{bmatrix} u_k^1 \\ \vdots \\ u_k^J \end{bmatrix} \quad y_k = \begin{bmatrix} y_k^1 \\ \vdots \\ y_k^J \end{bmatrix} \quad v_d = \begin{bmatrix} v_1 \\ \vdots \\ v_J \end{bmatrix} \quad R_d = \begin{bmatrix} R^1 & & 0 \\ & \ddots & \\ 0 & & R^J \end{bmatrix} > 0$$

$$\begin{cases} x_{k+1} = Ax_k + \sum_{j=1}^{J} B^j u_k^j + w \\ y_k^j = C^j x_k + v^j \quad (j = 1, 2, \cdots, J) \end{cases} \qquad \phi = \sum_{k=0}^{N} \left[x_k^T Q x_k + \sum_{j=1}^{J} u_k^{jT} R^j u_k^j \right]$$

$$\begin{cases} x_{k+1} = Ax_k + B_d u_k + w \\ y_k = C_d x_k + v_d \end{cases} \qquad \phi = \sum_{k=0}^{N} (x_k^T Q x_k + u_k^T R_d u_k)$$

Description of each equation is the same as the case of Centralized Control.

Design of Controller

$$\begin{cases} x_{k+1} = Ax_k + B_d u_k + w \\ y_k = C_d x_k + v_d \end{cases} \qquad \phi = \sum_{k=0}^{N} (x_k^T Q x_k + u_k^T R_d u_k)$$

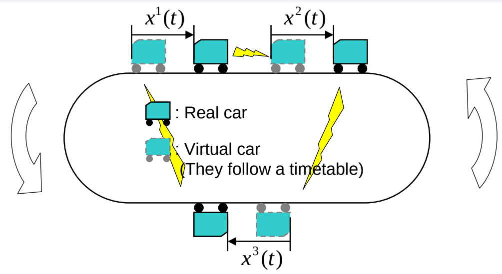
$$K = -(R_d + B_d^T P B_d)^{-1} B_d^T P A = \begin{bmatrix} K^1 \\ \vdots \\ K^J \end{bmatrix}$$

$$P = A^T P A + Q - A^T P B_d (R_d + B_d^T P B_d)^{-1} B_d^T P A \qquad P \geq 0$$

Algebraic Riccati equation

Optimal control : $u_k = K\hat{x}_k = \begin{bmatrix} K^1 \hat{x}_k \\ \vdots \\ K^J \hat{x}_k \end{bmatrix} = \begin{bmatrix} u_k^1 \\ \vdots \\ u_k^J \end{bmatrix}$

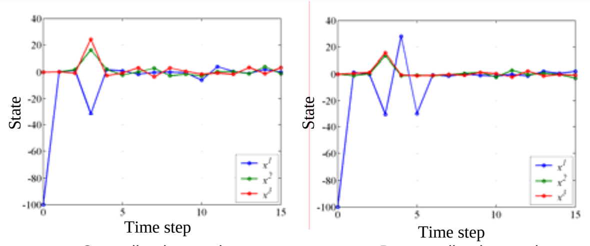
Decentralized Control - Example



Desired: errors converge to 0

Settings are same as the case of Centralized Control.

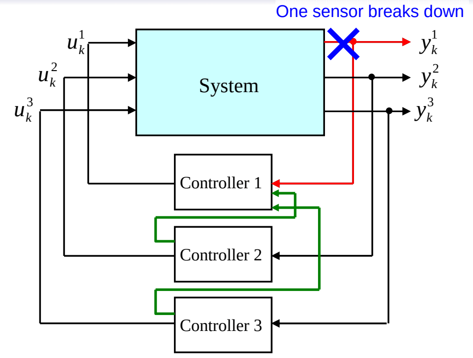
Simulation Result - Decentralized Control



Centralized control　　Decentralized control

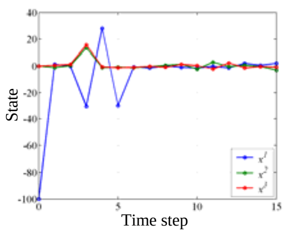
The result of decentralized control resemble those of centralized control

Simulation

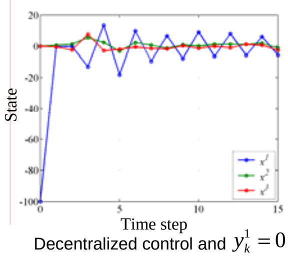


Controller 1 estimate a state using the information on other controllers

Simulation - One sensor break down



Decentralized control



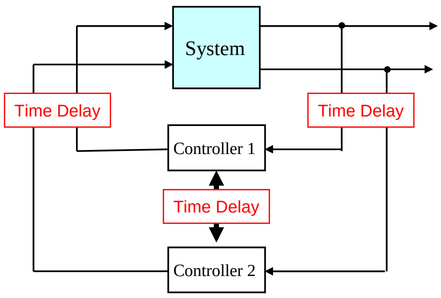
Decentralized control and $y_k^1 = 0$

The state also converged to 0 by the case of $y_k^1 = 0$

Conclusion

- Optimal control is given by state feedback.
- The state was estimated by Kalman Filter.
- When having a non-time-delay information structure, a controller can be designed like centralized control.
- Even when one sensor breaks down, controller can estimate a state and has controlled the system.

Future Work



- Decentralized state estimation problem
- Since a network system has time delays in many cases, we need to take them into consideration. [R1]

Bibliography

1. [B1]
池田, 吉川, 清水, 薦田, 青沼, 中山, 田村共著:
大規模システム -モデリング･制御･意思決定-,
昭晃堂, 1986

2. [B2]
Dimitri P. Bertsekas:
Dynamic Programming and Optimal Control,
Volume I, Second Edition,
Athena Scientific, Belmont, Massachusetts, 2000

3. [B3]
萩原朋道:
ディジタル制御入門,
コロナ社, 1999.

Bibliography

4. [B4]
片山徹:
システム同定 –部分空間法からのアプローチ-,
朝倉書店, 2004.

5. [B5]
有本卓:
カルマン･フィルター,
産業図書, 1977.

References

1. [R1]
Johan Nilsson, Bo Bernhardsson, Björn Wittenmark:
Stochastic Analysis and Control of Real-time Systems
with Random Time Delays,
Automatica, Vol. 34, No. 1, pp57-64, 1998.

Danke schön!!