Event-based LQG Control over Networks with Random Transmission Delays and Packet Losses

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Abstract: In Networked Control Systems (NCS), data networks not only limit the amount of information exchanged by system components but are also subject to stochastic packet delays and losses. In this paper, we present a controller that simultaneously addresses these problems by combining event-based and sequence-based control methods. At every time step, the proposed controller calculates a sequence of predicted control inputs and based on the expected future LQG costs decides whether it transmits the control sequence to the actuator. The proposed controller is evaluated with simulations.

Keywords: Event-based control, sequence-based control, LQG, networked control systems, NCS, stochastic control

1. INTRODUCTION

Control systems where the communication between system components is provided by digital data networks are referred to as Networked Control Systems (NCS). While the utilization of networks instead of direct point-to-point connections allows for flexible system architectures and reduces installation and maintenance costs, it can degrade system performance or even destabilize the system. Network impacts on system performance are, among others, (i) communication constraints that limit the amount of information that can be exchanged between the components of the control loop and (ii) stochastic delays and losses experienced by the transmitted information (Zhang et al. (2001), Hespanha et al. (2007), Antsaklis and Baillieul (2007)).

An approach to address problem (i) is to reduce the information amount communicated by the components of a control loop. This can be achieved by an event-based (also referred to as event-triggered) control philosophy (Hespanha et al. (2007)). This technique has been studied, e.g., in Åström and Bernhardsson (2002), Cogill (2009), Lehmann and Lunze (2012), Heemels and Donkers (2013). The key idea of event-based control is to transmit control inputs and/or measurements only when a certain event occurs. In the sensor-controller link, transmissions of periodically sampled measurements are triggered for example if the difference between the previously transmitted measurement and current measurement exceeds a certain level or if the estimation error covariance of the system state violates a tolerable threshold (Trimpe and D'Andrea (2012)). In the controller-actuator link, new control inputs are only transmitted to the actuator if the previously transmitted control sequences do not provide sufficient control quality. The actuator then holds the control inputs until new control inputs arrive (Heemels and Donkers (2013)).

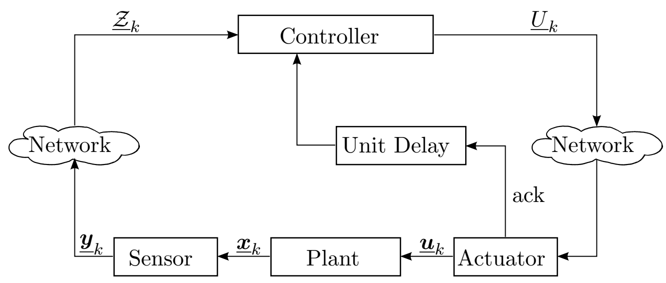

Fig. 1. Considered networked control system.

Problem (ii), i.e., network induced stochastic packet delays and losses, can be mitigated by a sequence-based control approach first mentioned in Bemporad (1998). The main idea of sequence-based control is to let the controller generate a sequence of predicted control inputs instead of a single control input and transmit the complete sequence to the actuator. The actuator can then apply control inputs from already received control sequences if a more recent control sequence is delayed or lost (see Gupta et al. (2006), Hekler et al. (2012), Fischer et al. (2013)).

The simultaneous treatment of the problems (i) and (ii) in the controller-actuator link constitutes the central idea of the proposed control strategy.

1.1 Related Work

Related results in the field of stochastic sequence-based control are, e.g., described in Gupta et al. (2006), Schenato et al. (2007), Quevedo et al. (2007), Hekler et al. (2012), Fischer et al. (2013), and the references therein. By applying the separation principle, an optimal sequence-based LQG controller for a NCS with lossy networks was derived in Gupta et al. (2006). The optimal sequence-based LQG controller with extension to unbounded stochastic delays, that constitutes the basis for the proposed solution, was derived in Fischer et al. (2013). This approach models the NCS as a Markovian Jump Linear System (MJLS) without network as presented in Hekler et al. (2012). The transition probabilities of the MJLS are defined by the network properties. The optimal closed-loop controller is then derived by means of stochastic dynamic programming. Furthermore, it is proved that separation also holds in presence of stochastic delays. The approaches in Gupta et al. (2006) and Fischer et al. (2013) assume that the actuator acknowledges successful transmissions. Networked sequence-based control of a deterministic nonlinear system is presented in Grune et al. (2009), where stochastic networks with packet delays and losses connect the sensor and the controller and the controller and the actuator. Instead of providing acknowledgments of successful transmissions, the actuator informs the controller about network failures.

Recent research on event-based control is described, e.g., in Varutti et al. (2009), Lunze and Lehmann (2010), Lehmann and Lunze (2012), Garcia and Antsaklis (2013), and Heemels and Donkers (2013). Event-based networked model predictive control of a deterministic nonlinear plant is discussed in Varutti et al. (2009), where the term event-based refers to the fact that the measurements do not arrive at the controller periodically due to delays and losses in the network. Because of this, the recalculation intervals of the control inputs are not fixed. In Garcia and Antsaklis (2013), an event-based control strategy is presented for an NCS where a network with negligible communication delays connects the sensor and the controller but the controller and the actuator are connected directly. The sensor transmits its local state estimate only when the difference between the local estimation and the model based prediction performed by the controller exceeds a certain threshold. The controller uses a system model to interpolate the system state between two consequent sensor transmissions. Another event-based control scheme with an ideal network connecting the sensor and the controller and no disturbances in the system model is presented in Lunze and Lehmann (2010). The communication frequency adapts to the desired system performance. This approach was then extended in Lehmann and Lunze (2012) to be able to cope with packet delays and losses. However only the sensor-controller link is considered. Heemels and Donkers (2013) proposes an observer-based control scheme for linear systems where measurements are sampled periodically but the transmissions to the controller are event-based. The model-based controller is able to generate control sequences that are then transmitted to the actuator if necessary. The networks are assumed ideal, i.e. no packet delays or losses occur.

1.2 Contribution

In this paper, we consider the system setup depicted in Fig. 1 with networks providing connections between the controller and the actuator and between sensor and the controller. In contrast to our former work, we not only address the problem of the compensation for stochastic delays and losses of the information communicated in the controller-actuator link that employs a TCP-like [1] network, but also address the problem of minimizing the traffic in this link. For this purpose, we combine the event-based and the sequence-based control methodologies. In particular, we extend the optimal sequence-based controller presented in Fischer et al. (2013) by including transmission costs in the LQG cost function. As a result, the controller only sends a control sequence when it sufficiently increases the control performance.

1.3 Outline

The remainder of the paper is organized as follows: In Sec. 2, we describe the considered system setup and formulate the problem. In Sec. 3, the main idea of the proposed solution is presented. The controller is derived in Sec. 4 and evaluated in Sec. 5. The paper is concluded by a summary and an outlook to future work.

1.4 Notation

We use the following notation: A random variable $\boldsymbol{x}$ is written in bold face letters, whereas a deterministic variable x is in normal lettering. The expectation operator is denoted by $\mathrm{E}\{\cdot\}$. The notation $\boldsymbol{x} \sim f(x)$ indicates that the random variable $\boldsymbol{x}$ is described by its probability density function $f(x)$. A vector-valued quantity $\underline{x}$ is highlighted by underlining. A matrix $\mathbf{A}$ is written in a bold face capital letter and the term $\mathbf{A}^\dagger$ denotes the Moore-Penrose pseudo-inverse of $\mathbf{A}$. The notation x_k refers to the quantity x at time step k. The set $\{x_a, x_{a+1}, \ldots, x_b\}$ is abbreviated by $x_{a:b}$. Finally, $x_{k|t}$ denotes the quantity x at time step k based on the information available up to time t.

2. SYSTEM SETUP & PROBLEM FORMULATION

We consider the system depicted in Fig. 1, whose components are time-triggered and synchronized. The state space representation of the system is given by

$$\begin{aligned} \underline{\boldsymbol{x}}_{k+1} &= \mathbf{A}\underline{\boldsymbol{x}}_k + \mathbf{B}\underline{u}_k + \underline{\boldsymbol{w}}_k \ , \\ \underline{\boldsymbol{y}}_k &= \mathbf{C}\underline{\boldsymbol{x}}_k + \underline{\boldsymbol{v}}_k \ , \end{aligned} \tag{1}$$

with $\underline{\boldsymbol{x}}_k \in \mathbb{R}^n$ denoting the system state at time step k, $\underline{u}_k \in \mathbb{R}^m$ the control input applied to the plant by the actuator, and $\underline{\boldsymbol{y}}_k \in \mathbb{R}^q$ the measured output. The matrices $\mathbf{A} \in \mathbb{R}^{n\times n}$, $\mathbf{B} \in \mathbb{R}^{n\times m}$, and $\mathbf{C} \in \mathbb{R}^{q\times n}$ are known. The terms $\underline{\boldsymbol{w}}_k \sim f^w(\underline{w}_k)$ and $\underline{\boldsymbol{v}}_k \sim f^v(\underline{v}_k)$ represent mutually independent, stationary, zero-mean, discrete-time Gaussian processes with covariances

$$\mathbf{W} = \mathrm{E}\left\{\underline{\boldsymbol{w}}_k\underline{\boldsymbol{w}}_k^T\right\} \quad \text{and} \quad \mathbf{V} = \mathrm{E}\left\{\underline{\boldsymbol{v}}_k\underline{\boldsymbol{v}}_k^T\right\} \ . \tag{2}$$

The initial state $\underline{x}_0$ is independent of other random variables and has a Gaussian distribution with

$$\hat{\underline{x}}_0 = \mathrm{E}\left\{\underline{\boldsymbol{x}}_0\right\} \text{ and } \mathbf{X}_0 = \mathrm{E}\left\{\left(\underline{\boldsymbol{x}}_0 - \hat{\underline{x}}_0\right)\left(\underline{\boldsymbol{x}}_0 - \hat{\underline{x}}_0\right)^T\right\} \ . \tag{3}$$

The information exchange between the controller and the actuator (CA-link), as well as between the sensor and the controller (SC-link) is provided by packet-based digital networks that are capable of transmitting large time-stamped data packets. The maximum size of a data packet

[1] The term "TCP-like" refers to an idealized model of a real TCP/IP. The TCP-like network provides instantaneous acknowledgments for successful transmissions. This assumption is restrictive, however it can be maintained if acknowledgments are prioritized over residual traffic. Furthermore, results that are obtained for TCP-like networks are of theoretical interest because they can serve as a basis for understanding fundamental properties of NCS.

is assumed to be larger than the size of a single control input, which is usually the case for, e.g., TCP/IP networks.

Furthermore, the networks suffer from stochastic time-varying delays and packet losses. The delays are described by stationary random processes $\boldsymbol{\tau}_k^{CA} \in \mathbb{N}_0$ and $\boldsymbol{\tau}_k^{SC} \in \mathbb{N}_0$ with known probability density functions $f^{CA}\left(\tau_k^{CA}\right)$ and $f^{SC}\left(\tau_k^{SC}\right)$. The processes $\boldsymbol{\tau}_k^{CA}$ and $\boldsymbol{\tau}_k^{SC}$ specify how many time steps a data packet will be delayed if sent at time step k. Packet losses correspond to infinite time delays and, therefore, are naturally integrated in this description.

In addition, the controller-actuator network (CA-link) implements a TCP-like protocol, i.e., when a packet arrives at the actuator, the controller receives an instantaneous acknowledgment. The sensor-controller network (SC-Link) does not require to employ a TCP-like protocol. It can, e.g., implement a so-called UDP-like protocol that does not acknowledge successful transmissions.

We consider the case where the actuator does not have enough computational resources to compute a control input autonomously. If no control packet is received by the actuator (due to losses or delays in the communication link or because no packet was sent by the controller), the actuator has several options to choose a control input. The most common strategies are: to apply the previous control input again (hold strategy), to apply a known fixed control input, e.g., zero (zero input strategy), or to apply a control input transmitted by the controller as part of an earlier control packet (sequence-based strategy). Our proposed control method will use the sequence-based strategy that is explained more detailed in Sec. 3.

The performance of the system is measured in terms of the receding horizon Linear Quadratic Gaussian (LQG) cost function J_0 given by

$$J_0 = \mathrm{E}\left\{ \underline{\boldsymbol{x}}_K^T \mathbf{Q}_K \underline{\boldsymbol{x}}_K + \sum_{k=0}^{K-1} \left(\underline{\boldsymbol{x}}_k^T \mathbf{Q}_k \underline{\boldsymbol{x}}_k + \underline{u}_k^T \mathbf{R}_k \underline{u}_k \right) \middle| \mathcal{I}_0 \right\} , \tag{4}$$

where $K \in \mathbb{N}$ denotes the horizon length, $\mathbf{Q}_k$ is positive semi-definite and $\mathbf{R}_k$ positive definite. The term $\mathcal{I}_0$ is the set of information available to the controller at time step $k = 0$. It will formally be defined in Sec. 4.

In this paper, we are interested in finding a controller that on the one hand leads to a good performance (indicated by (4)) and, in particular, compensates for the network-induced effects but on the other hand reduces the network traffic to a minimum and only sends data to the actuator if this significantly improves the performance. In this work, we restrict ourselves to the event-based scheme in the CA-Link. The event-based measurement transmission in the SC-link will be adopted in future work.

3. PROPOSED SOLUTION

The key element of the proposed solution is to use a sequence-based design philosophy. A sequence-based controller not only generates a single control input per time step but also a sequence of predicted control inputs applicable at future time steps. The control inputs are dispatched in a data packet and sent to the actuator, which stores the control sequence in a buffer and applies one or more of control inputs taken from that sequence if a more recent packet fails to arrive. Therefore, the sequence-based mechanism is not only suitable to compensate for either network induced time delays or packet losses in the CA-link (Gupta et al. (2006), Hekler et al. (2012), Fischer et al. (2013)) or to reduce the communication between controller and actuator (Heemels and Donkers (2013)) but also for a combination of both.

To balance the cost of a data transmission with the expected benefit in performance measured by (4), we extend the cost function (4) by the term $\boldsymbol{s}_k S$ with $\boldsymbol{s}_k \in \{0, 1\}$ and $S \geq 0$ that represents the transmission cost such that

$$J_0 = \mathrm{E}\left\{ \underline{\boldsymbol{x}}_K^T \mathbf{Q}_K \underline{\boldsymbol{x}}_K + \sum_{k=0}^{K-1} \left(s_k S + \underline{\boldsymbol{x}}_k^T \mathbf{Q}_k \underline{\boldsymbol{x}}_k + \underline{\boldsymbol{u}}_k^T \mathbf{R}_k \underline{\boldsymbol{u}}_k \right) \middle| \mathcal{I}_0 \right\} . \tag{5}$$

The transmission costs incur if the controller sends a data packet $\underline{U}_k$ to the actuator ($s_k = 1$) and do not incur if no packet is sent ($s_k = 0$). Similar to $\mathbf{Q}_k$ and $\mathbf{R}_k$, the transmission cost parameter S is a design parameter and has to be chosen according to a desired system behavior.

In this work, we address the problem of the minimization of (5). Since the optimal solution to this problem would require the evaluation of an exponentially growing decision tree, we propose a suboptimal sequence-based controller that performs following steps:

(1) In the first step, the controller computes a control sequence candidate that would optimally compensate for delays and losses if transmitted to the actuator.
(2) In a second step, the controller decides whether the expected increase in performance is sufficiently high to justify a data transmission. If this is the case, the control packet is sent, otherwise it is discarded.

The derivation of this event-based and sequence-based controller is described in the next section.

4. DERIVATION OF THE CONTROLLER

To derive the event-based and sequence-based controller, we first reformulate the considered NCS as a Markov Jump Linear System (MJLS) in Sec. 4.1. Based on this model, we show how to calculate a control packet candidate in Sec. 4.2. The decision rule whether to send the control packet candidate or not is presented in Sec. 4.3.

4.1 System reformulation

As mentioned above, a control packet $\underline{U}_k$ consists of a sequence of control inputs. For the rest of the paper we denote the entries of such a control sequence by $\underline{u}_{k+i|k}$ with $i \in \{0, 1, ..., N\}, N \in \mathbb{N}_0$, where the index specifies that the control input $\underline{u}_{k+i|k}$ was generated at time step k and is intended to be used at time step $k + i$. Thus, the control sequence $\underline{U}_k$ has the length $N + 1$ and can be written as

$$\underline{U}_k = \left[\underline{u}_{k|k}^T \ \underline{u}_{k+1|k}^T \ \cdots \ \underline{u}_{k+N|k}^T \right]^T . \tag{6}$$

With this notation, the control input $\underline{u}_k$ applied by the actuator can be described by

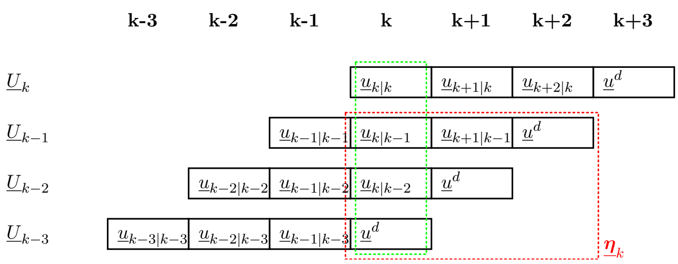

Fig. 2. Temporally aligned control sequences $\underline{U}_{k-3}, \ldots, \underline{U}_k$. At time step k, one of the control inputs $\underline{u}_{k|k}, \underline{u}_{k|k-1}, \underline{u}_{k|k-2}$, or $\underline{u}^d$, indicated by the green rectangle, can be applied.

$$\underline{u}_k = \begin{cases} \underline{u}_{k|k-\boldsymbol{\theta}_k}, & \text{if } \boldsymbol{\theta}_k < N+1 \\ \underline{u}^d, & \text{if } \boldsymbol{\theta}_k = N+1 \end{cases} , \tag{7}$$

where $\underline{u}^d$ is a default control input, that is applied by the actuator if it runs out of buffered control inputs. The term $\boldsymbol{\theta}_k \in \{0, 1, \ldots, N+1\}$ is a switching process, that indicates the age of the buffered control sequence (see Sec. 3 in Fischer et al. (2013) for more details). For the case where the control sequence $\underline{U}_k$ is transmitted at every time step, i.e., $s_{0:k} = 1$, it has been shown in Hekler et al. (2012), Fischer et al. (2012), and Fischer et al. (2013) that the switching process can be described as a state of a Markov chain with transition matrix $\mathbf{T}$ given by

$$T = \begin{bmatrix} p_{00} & p_{01} & 0 & 0 & \ldots & 0 \\ p_{10} & p_{11} & p_{12} & 0 & \ldots & 0 \\ p_{20} & p_{21} & p_{22} & p_{23} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & p_{(r-1)(r)} \\ p_{r0} & p_{r1} & p_{r2} & p_{r3} & \ldots & p_{rr} \end{bmatrix} \tag{8}$$

with

$$p_{ij} = \text{Prob}\left(\boldsymbol{\theta}_{k+1} = j \middle| \boldsymbol{\theta}_k = i\right), \quad r = N+1 .$$

This transition matrix only depends on the probability density function $f^{CA}(\tau_k)$ of the stochastic process $\boldsymbol{\tau}_k^{CA}$, that models the transmission delays of the control sequences.

In our case, where it is possible that no control sequence has been sent at any previous time step, the transition matrix at time step k not only depends on $\boldsymbol{\tau}_k^{CA}$, but also on the transmission history $s_{0:k-1}$. Fortunately, the influence of the transmission history on the state of the Markov chain $\boldsymbol{\theta}_k$ is limited because if a control sequence is not sent, this only influences the transition matrix of the Markov chain for the N subsequent time steps. Consequently, the transition probabilities of $\boldsymbol{\theta}_k$ only depend on the finite transmission history $\boldsymbol{s}_{k-N-1:k-1}$ and we obtain a set of 2^N possible transition matrices from which exactly one is active at a certain time step. The set of transition matrices is denoted by $\mathbf{T}^{\mathcal{S}_k}$ where $\mathcal{S}_k = s_{k-N-1:k-1}$ is any possible realization of the sequence $s_{k-N-1:k-1}$. The elements of $\mathbf{T}^{\mathcal{S}_k}$ can be calculated similarly to Fischer et al. (2013) but with time-varying state set of the Markov chain, i.e., a state of the Markov chain does not exist at time step $k-1$ if the according control sequence was not sent. Therefore, the calculation of p_{ji} is given by

$$p_{ji} = \begin{cases} 0, & \text{for } i \geq j+2 \\ \left(1 - \sum\limits_{r=0}^{i} q_r\right) \prod\limits_{\substack{l=i \\ i>1}}^{N-1} \left(s_{k-l} + s_{k-l} \dfrac{1 - \sum_{r=0}^{l-1} q_r}{1 - \sum_{r=0}^{l} q_r} \right), & \text{for } i = j+1 \\ q_i \prod\limits_{\substack{l=i \\ i>1}}^{N-1} \left(s_{k-l} + s_{k-l} \dfrac{1 - \sum_{r=0}^{l-1} q_r}{1 - \sum_{r=0}^{l} q_r} \right), & \text{for } i \leq j < N \\ \left(1 - \sum\limits_{r=0}^{N} q_r\right) \prod\limits_{\substack{l=i \\ i>1}}^{N-1} \left(s_{k-l} + s_{k-l} \dfrac{1 - \sum_{r=0}^{l-1} q_r}{1 - \sum_{r=0}^{l} q_r} \right), & \text{for } i = j = N , \end{cases}$$

where q_r denotes the probability that the control sequence will be delayed for $r \in \mathbb{N}_0$ time steps. This probability can be derived from $f^{CA}(\tau_k)$.

Having obtained the properties of the switching process, we can model the situation at the actuator by the following stochastic state space equation

$$\begin{aligned} \underline{\eta}_{k+1} &= \mathbf{F}\underline{\eta}_k + \mathbf{G}\underline{U}_k \\ \underline{\boldsymbol{u}}_k &= \mathbf{H}_{\boldsymbol{\theta}_k}\underline{\eta}_k + \mathbf{J}_{\boldsymbol{\theta}_k}\underline{U}_k \end{aligned} , \tag{9}$$

with the state vector

$$\underline{\eta}_k = \begin{pmatrix} \left[\underline{u}_{k|k-1}^T \; \underline{u}_{k+1|k-1}^T \; \cdots \; \underline{u}_{k+N-1|k-1}^T\right]^T \\ \left[\underline{u}_{k|k-2}^T \; \underline{u}_{k+1|k-2}^T \; \cdots \; \underline{u}_{k+N-2|k-2}^T\right]^T \\ \vdots \\ \underline{u}_{k|k-N} \\ \underline{u}^d \end{pmatrix} , \tag{10}$$

that contains all control inputs from sequences $\underline{U}_{k-1}, \ldots, \underline{U}_{k-N}$ whose control inputs still could be applied to the plant in the following time steps and the default control input $\underline{u}^d$ (Fig. 2 illustrates $\underline{\eta}_k$ for $N = 2$) and

$$\mathbf{F} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \ldots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{I} \end{bmatrix}, \qquad \begin{aligned} \mathbf{G} &= \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \\ \mathbf{J}_{\boldsymbol{\theta}_k} &= \left[\delta_{(\boldsymbol{\theta}_k,0)}\mathbf{I} \; \mathbf{0}\right], \end{aligned}$$

$$\mathbf{H}_{\boldsymbol{\theta}_k} = \left[\delta_{(\boldsymbol{\theta}_k,1)}\mathbf{I} \; \mathbf{0} \; \delta_{(\boldsymbol{\theta}_k,2)}\mathbf{I} \; \mathbf{0} \; \ldots \delta_{(\boldsymbol{\theta}_k,N)}\mathbf{I}\right],$$

$$\delta_{(\boldsymbol{\theta}_k,i)} = \begin{cases} 1, & \text{if } \boldsymbol{\theta}_k = i \\ 0, & \text{if } \boldsymbol{\theta}_k \neq i \end{cases},$$

where $\mathbf{0}$ is the matrix with all elements equal to zero and $\mathbf{I}$ the identity matrix of appropriate dimensions. It can be seen from (9) that the actual control input $\underline{\boldsymbol{u}}_k$ is selected from $\underline{\eta}_k$ and $\underline{U}_k$ according to the state of the Markov chain $\boldsymbol{\theta}_k$, i.e., depending on the buffered control sequence.

Combining (1) and (9) and introducing the augmented state

$$\underline{\boldsymbol{\xi}}_k = \left[\underline{\boldsymbol{x}}_k^T \; \underline{\eta}_k^T\right]^T , \tag{11}$$

we can express the considered system as a Markovian Jump Linear System (MJLS) with jumping parameter $\boldsymbol{\theta}_k$ according to

$$\underline{\boldsymbol{\xi}}_{k+1} = \overbrace{\begin{bmatrix} \mathbf{A} & \mathbf{B}\mathbf{H}_{\boldsymbol{\theta}_k} \\ \mathbf{0} & \mathbf{F} \end{bmatrix}}^{\widetilde{\mathbf{A}}_k} \underline{\boldsymbol{\xi}}_k + \overbrace{\begin{bmatrix} \mathbf{B}\mathbf{J}_{\boldsymbol{\theta}_k} \\ \mathbf{G} \end{bmatrix}}^{\widetilde{\mathbf{B}}_k} \underline{U}_k + \begin{pmatrix} \underline{\boldsymbol{w}}_k \\ \underline{0} \end{pmatrix} = \widetilde{\mathbf{A}}_k \underline{\boldsymbol{\xi}}_k + \widetilde{\mathbf{B}}_k \underline{U}_k + \underline{\widetilde{\boldsymbol{w}}}_k \ . \tag{12}$$

4.2 Computation of the control sequence candidate

The optimal control sequence candidate is the solution of the minimization of (5) over all possible control inputs and under consideration of all possibilities of $\boldsymbol{s}_{k:K}$, i.e., all combinations of future control sequences being transmitted or not. Since no analytic solution for this decision problem is known, we approximate the problem and derive the control sequence candidate at time step k under the assumption that the controller will send all subsequent control packets of the current control sequence candidate, i.e., $s_{k:K} = 1$. Of course, this is only an approximation since the controller can decide not to send future control sequences as well.

To provide a consistent notation, we first express the stage costs (5) in terms of the augmented state $\underline{\boldsymbol{\xi}}_k$

$$J_0 = \mathrm{E}\left\{ \underline{\boldsymbol{\xi}}_K^T \widetilde{\mathbf{Q}}_K \underline{\boldsymbol{\xi}}_K + \sum_{k=0}^{K-1} \left(\boldsymbol{s}_k S + \underline{\boldsymbol{\xi}}_k^T \widetilde{\mathbf{Q}}_k \underline{\boldsymbol{\xi}}_k + \boldsymbol{s}_k \underline{U}_k^T \widetilde{\mathbf{R}}_k \underline{U}_k \right) \middle| \mathcal{I}_0 \right\} \ , \tag{13}$$

with minimal costs-to-go J_k^* at time step k given by

$$J_K^* = \mathrm{E}\left\{ \underline{\boldsymbol{\xi}}_K^T \widetilde{\mathbf{Q}}_K \underline{\boldsymbol{\xi}}_K \middle| \mathcal{I}_K \right\} \ ,$$
$$J_k^* = \min_{\underline{U}_k} \mathrm{E}\left\{ \boldsymbol{s}_k S + \underline{\boldsymbol{\xi}}_k^T \widetilde{\mathbf{Q}}_k \underline{\boldsymbol{\xi}}_k + \boldsymbol{s}_k \underline{U}_k^T \widetilde{\mathbf{R}}_k \underline{U}_k + J_{k+1}^* \middle| \mathcal{I}_k \right\} \ , \tag{14}$$

where

$$\widetilde{\mathbf{Q}}_K = \begin{bmatrix} \mathbf{Q}_K & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \ , \quad \widetilde{\mathbf{Q}}_k = \begin{bmatrix} \mathbf{Q}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_k^T \mathbf{R}_k \mathbf{H}_k \end{bmatrix} \ , \quad \widetilde{\mathbf{R}}_k = \mathbf{J}_k^T \mathbf{R}_k \mathbf{J}_k \ . \tag{15}$$

Now we can minimize (13) by the means of dynamic programming with the information set available to the controller at time step k. The latter is given by

$$\mathcal{I}_k = \left\{ \mathcal{Z}_{0:k}, \boldsymbol{\theta}_{0:k-1}, \underline{U}_{0:k-1}, \boldsymbol{s}_{0:k-1} \right\} \ , \tag{16}$$

where $\underline{U}_{0:k-1}$ denotes the previously computed control inputs, the variables $\boldsymbol{s}_{0:k-1} \in \{0,1\}$ indicate which of these control packets $\underline{U}_{0:k-1}$ were actually transmitted, the variables $\boldsymbol{\theta}_{0:k-1}$ specify which control packets have been received by the actuator (based on the acknowledgment signals) and the term $\mathcal{Z}_{0:k}$ denotes the set of received measurements up to time step k.

At time step K, the minimal costs-to-go are

$$J_K^* = \mathrm{E}\left\{ \underline{\boldsymbol{\xi}}_K^T \widetilde{\mathbf{Q}}_K \underline{\boldsymbol{\xi}}_K \middle| \mathcal{I}_k \right\} = \mathrm{E}\left\{ \underline{\boldsymbol{\xi}}_K^T \mathbf{P}_K \underline{\boldsymbol{\xi}}_K \middle| \mathcal{I}_k \right\} \ , \tag{17}$$

with $\mathbf{P}_K = \widetilde{\mathbf{Q}}_K$.

When we assume that the controller will send a control sequence at time step $K-1$ the minimal costs-to-go J_{K-1}^* can be calculated according to

$$\begin{aligned} J_{K-1}^* &= \min_{\underline{U}_{K-1}} \mathrm{E}\left\{ S + \underline{\boldsymbol{\xi}}_{K-1}^T \widetilde{\mathbf{Q}}_{K-1} \underline{\boldsymbol{\xi}}_{K-1} + \underline{U}_{K-1}^T \widetilde{\mathbf{R}}_{K-1} \underline{U}_{K-1} + J_K^* \middle| \mathcal{I}_{K-1} \right\} \\ &= \mathrm{E}\left\{ \underline{\boldsymbol{\xi}}_{K-1}^T \left(\widetilde{\mathbf{Q}}_{K-1} + \widetilde{\mathbf{A}}_{K-1}^T \mathbf{P}_K \widetilde{\mathbf{A}}_{K-1} \right) \underline{\boldsymbol{\xi}}_{K-1} \middle| \mathcal{I}_{K-1} \right\} \\ &+ \min_{\underline{U}_{K-1}} \Big[\underline{U}_{K-1}^T \mathrm{E}\left\{ \widetilde{\mathbf{R}}_{K-1} + \widetilde{\mathbf{B}}_{K-1}^T \mathbf{P}_K \widetilde{\mathbf{B}}_{K-1} \middle| \mathcal{I}_{K-1} \right\} \underline{U}_{K-1} \\ &+ 2 \cdot \mathrm{E}\left\{ \underline{\boldsymbol{\xi}}_{K-1}^T \middle| \mathcal{I}_{K-1} \right\} \mathrm{E}\left\{ \widetilde{\mathbf{A}}_{K-1}^T \mathbf{P}_K \widetilde{\mathbf{B}}_{K-1} \middle| \mathcal{I}_{K-1} \right\} \underline{U}_{K-1} \Big] \\ &+ S + \mathrm{E}\left\{ \underline{\widetilde{\boldsymbol{w}}}_{K-1}^T \mathbf{P}_K \underline{\widetilde{\boldsymbol{w}}}_{K-1} \middle| \mathcal{I}_{K-1} \right\} \ . \end{aligned} \tag{18}$$

The differentiation of (18) with respect to $\underline{U}_{k-1}$ and setting to zero yields

$$\underline{U}_{K-1} = -\left(\mathrm{E}\left\{ \widetilde{\mathbf{R}}_{K-1} + \widetilde{\mathbf{B}}_{K-1}^T \mathbf{P}_K \widetilde{\mathbf{B}}_{K-1} \middle| \mathcal{I}_{K-1} \right\} \right)^{\dagger} \times \mathrm{E}\left\{ \widetilde{\mathbf{B}}_{K-1}^T \mathbf{P}_K \widetilde{\mathbf{A}}_{K-1} \middle| \mathcal{I}_{K-1} \right\} \mathrm{E}\left\{ \underline{\boldsymbol{\xi}}_{K-1} \middle| \mathcal{I}_{K-1} \right\} \ . \tag{19}$$

The Moore-Penrose pseudo-inverse is used for convenience in order to avoid a reformulation of the optimization problem for control inputs that are applied with zero probability. For detailed description see Fischer et al. (2013).

Using (19) in (18) yields

$$\begin{aligned} J_{K-1}^* &= \mathrm{E}\left\{ \underline{\boldsymbol{\xi}}_{K-1}^T \mathbf{P}_{K-1} \underline{\boldsymbol{\xi}}_{K-1} \middle| \mathcal{I}_{K-1} \right\} \\ &+ \mathrm{E}\left\{ \underline{\boldsymbol{\mathcal{E}}}_{K-1}^T \mathbf{\Xi}_{K-1} \underline{\boldsymbol{\mathcal{E}}}_{K-1} \middle| \mathcal{I}_{K-1} \right\} \\ &+ \mathrm{E}\left\{ \underline{\widetilde{\boldsymbol{w}}}_{K-1}^T \mathbf{P}_K \underline{\widetilde{\boldsymbol{w}}}_{K-1} \middle| \mathcal{I}_{K-1} \right\} + S \ , \end{aligned} \tag{20}$$

with the definition

$$\underline{\boldsymbol{\mathcal{E}}}_k = \underline{\boldsymbol{\xi}}_k - \mathrm{E}\left\{ \underline{\boldsymbol{\xi}}_k \middle| \mathcal{I}_k \right\} \ , \tag{21}$$

and

$$\begin{aligned} \mathbf{P}_{K-1} &= \mathrm{E}\left\{ \widetilde{\mathbf{Q}}_{K-1} + \widetilde{\mathbf{A}}_{K-1}^T \mathbf{P}_K \widetilde{\mathbf{A}}_{K-1} \middle| \mathcal{I}_{K-1} \right\} - \mathbf{\Xi}_{K-1} \\ \mathbf{\Xi}_{K-1} &= \mathrm{E}\left\{ \widetilde{\mathbf{A}}_{K-1}^T \mathbf{P}_K \widetilde{\mathbf{B}}_{K-1} \middle| \mathcal{I}_{K-1} \right\} \\ &\times \left(\mathrm{E}\left\{ \widetilde{\mathbf{R}}_{K-1} + \widetilde{\mathbf{B}}_{K-1}^T \mathbf{P}_K \widetilde{\mathbf{B}}_{K-1} \middle| \mathcal{I}_{K-1} \right\} \right)^{\dagger} \\ &\times \mathrm{E}\left\{ \widetilde{\mathbf{B}}_{K-1}^T \mathbf{P}_K \widetilde{\mathbf{A}}_{K-1} \middle| \mathcal{I}_{K-1} \right\} \ . \end{aligned} \tag{22}$$

If we proceed with the minimization assuming that the controller will transmit all future control sequences, it follows for the minimal cost-to-go J_k^* at time step k that

$$\begin{aligned} J_k^* &= \mathrm{E}\left\{ \underline{\boldsymbol{\xi}}_k^T \mathbf{P}_k \underline{\boldsymbol{\xi}}_k \middle| \mathcal{I}_k \right\} + \sum_{i=k}^{K-1} \mathrm{E}\left\{ \underline{\widetilde{\boldsymbol{w}}}_i^T \mathbf{P}_i \underline{\widetilde{\boldsymbol{w}}}_i \middle| \mathcal{I}_k \right\} \\ &+ \sum_{i=k}^{K-1} \mathrm{E}\left\{ \underline{\boldsymbol{\mathcal{E}}}_i^T \mathbf{\Xi}_i \underline{\boldsymbol{\mathcal{E}}}_i \middle| \mathcal{I}_k \right\} + \sum_{i=k}^{K-1} S \ . \end{aligned} \tag{23}$$

The expected values in (23) can be calculated explicitly by conditioning on $\boldsymbol{\theta}_{k-1} = j$, $\boldsymbol{s}_{k-1:k-N-1}$ and $\boldsymbol{s}_k = 1$. This is possible because $\boldsymbol{\theta}_{k-1}$ and $\boldsymbol{s}_{k-1:k-N-1}$ are included in $\mathcal{I}_k$. It holds

$$\underline{U}_k = -\left[\sum_{i=0}^{N+1} p_{ji}^{\mathcal{S}_k}\left(\widetilde{\mathbf{R}}_{|i} + \widetilde{\mathbf{B}}_{|i}^T \mathrm{E}\left\{\mathbf{P}_{k+1}|\,\mathcal{K}_{k+1}^i\right\}\widetilde{\mathbf{B}}_{|i}\right)\right]^\dagger \times \left[\sum_{i=0}^{N+1} p_{ji}^{\mathcal{S}_k}\widetilde{\mathbf{B}}_{|i}^T \mathrm{E}\left\{\mathbf{P}_{k+1}|\,\mathcal{K}_{k+1}^i\right\}\widetilde{\mathbf{A}}_{|i}\right]\mathrm{E}\left\{\underline{\boldsymbol{\xi}}_k\middle|\mathcal{I}_k\right\} ,$$

$$\begin{aligned}\mathrm{E}\left\{\mathbf{P}_k|\,\mathcal{K}_k^j\right\} &= \left[\sum_{i=0}^{N+1} p_{ji}^{\mathcal{S}_k}\left(\widetilde{\mathbf{Q}}_{|i} + \widetilde{\mathbf{A}}_{|i}^T \mathrm{E}\left\{\mathbf{P}_{k+1}|\,\mathcal{K}_{k+1}^i\right\}\widetilde{\mathbf{A}}_{|i}\right)\right] \\ &- \left[\sum_{i=0}^{N+1} p_{ji}^{\mathcal{S}_k}\widetilde{\mathbf{A}}_{|i}^T \mathrm{E}\left\{\mathbf{P}_{k+1}|\,\mathcal{K}_{k+1}^i\right\}\widetilde{\mathbf{B}}_{|i}\right] \\ &\times \left[\sum_{i=0}^{N+1} p_{ji}^{\mathcal{S}_k}\left(\widetilde{\mathbf{R}}_{|i} + \widetilde{\mathbf{B}}_{|i}^T \mathrm{E}\left\{\mathbf{P}_{k+1}|\,\mathcal{K}_{k+1}^i\right\}\widetilde{\mathbf{B}}_{|i}\right)\right]^\dagger \\ &\times \left[\sum_{i=0}^{N+1} p_{ji}^{\mathcal{S}_k}\widetilde{\mathbf{B}}_{|i}^T \mathrm{E}\left\{\mathbf{P}_{k+1}|\,\mathcal{K}_{k+1}^i\right\}\widetilde{\mathbf{A}}_{|i}\right] ,\end{aligned} \tag{24}$$

with the set $\mathcal{K}_{k+1}^i = \{\mathcal{I}_k, \mathcal{S}_k, \boldsymbol{\theta}_k = i, \boldsymbol{s}_k = 1\}$ and $\mathbf{X}_{|i}$ denoting a realization of a stochastic matrix for the case where $\boldsymbol{\theta}_k = i$. The probabilities $p_{ji}^{\mathcal{S}_k}$ denote the elements of the transition matrix $T^{\mathcal{S}_k}$.

Proof. The proof for (23) and (24) works similar to Fischer et al. (2013). □

4.3 Decision step

To decide whether the control sequence candidate $\underline{U}_k$ should be transmitted to the actuator, the controller has to analyze the expected costs-to-go for both cases: if it transmits $\underline{U}_k$ and if it does not. With J_k^* denoting the expected costs-to-go if $\underline{U}_k$ is sent and $\overline{J}_k^*$ denoting the expected costs-to-go if $\underline{U}_k$ is not sent, we can identify the decision rule as

- $J_k^* - \overline{J}_k^* \geq 0 \rightarrow$ do not send,
- $J_k^* - \overline{J}_k^* < 0 \rightarrow$ send. (25)

The value of J_k^* is given by (23) with $\mathbf{P}_i$ and $\boldsymbol{\Xi}_i$ defined by (24). The expected costs for not transmitting at the current time step and under assumption that all subsequent control sequences will be sent are given by

$$\begin{aligned}\overline{J}_k^* &= \mathrm{E}\left\{\underline{\boldsymbol{\xi}}_k^T\mathbf{P}_k\underline{\boldsymbol{\xi}}_k\middle|\overline{\mathcal{I}}_k\right\} + \sum_{i=k}^{K-1}\mathrm{E}\left\{\underline{\widetilde{\boldsymbol{w}}}_k^T\mathbf{P}_i\underline{\widetilde{\boldsymbol{w}}}_i\middle|\overline{\mathcal{I}}_k\right\} \\ &+ \sum_{i=k+1}^{K-1} S + \mathrm{E}\left\{\underline{\boldsymbol{\xi}}_k^T\boldsymbol{\Xi}_k\underline{\boldsymbol{\xi}}_k\middle|\overline{\mathcal{I}}_k\right\} \\ &+ \sum_{i=k+1}^{K-1}\mathrm{E}\left\{\overline{\underline{\boldsymbol{\mathcal{E}}}}_i^T\boldsymbol{\Xi}_i\overline{\underline{\boldsymbol{\mathcal{E}}}}_i\middle|\overline{\mathcal{I}}_k\right\} ,\end{aligned} \tag{26}$$

where $\overline{\mathcal{I}}_i$ with $i \in \{k, k+1, ..., K-1\}$ denotes the information set equal to $\mathcal{I}_i$ except for s_k being zero, and the estimation error $\overline{\underline{\boldsymbol{\mathcal{E}}}}_k$ is defined as

$$\overline{\underline{\boldsymbol{\mathcal{E}}}}_k = \underline{\boldsymbol{\xi}}_k - \mathrm{E}\left\{\underline{\boldsymbol{\xi}}_k\middle|\overline{\mathcal{I}}_k\right\} . \tag{27}$$

Using (23) and (26), the difference $J_k^* - \overline{J}_k^*$ in the decision rule can be written as

$$J_k^* - \overline{J}_k^* = \Delta_{con} + \Delta_{est} , \tag{28}$$

with

$$\begin{aligned}\Delta_{con} &= \mathrm{E}\left\{\underline{\boldsymbol{\xi}}_k^T\mathbf{P}_k\underline{\boldsymbol{\xi}}_k\middle|\mathcal{I}_k\right\} - \mathrm{E}\left\{\underline{\boldsymbol{\xi}}_k^T\mathbf{P}_k\underline{\boldsymbol{\xi}}_k\middle|\overline{\mathcal{I}}_k\right\} \\ &+ \sum_{i=k}^{K-1}\mathrm{E}\left\{\underline{\widetilde{\boldsymbol{w}}}_i^T\mathbf{P}_i\underline{\widetilde{\boldsymbol{w}}}_i\middle|\mathcal{I}_k\right\} - \sum_{i=k}^{K-1}\mathrm{E}\left\{\underline{\widetilde{\boldsymbol{w}}}_i^T\mathbf{P}_i\underline{\widetilde{\boldsymbol{w}}}_i\middle|\overline{\mathcal{I}}_k\right\} \\ &+ \mathrm{E}\left\{\underline{\boldsymbol{\mathcal{E}}}_k^T\boldsymbol{\Xi}_k\underline{\boldsymbol{\mathcal{E}}}_k\middle|\mathcal{I}_k\right\} - \mathrm{E}\left\{\underline{\boldsymbol{\xi}}_k^T\boldsymbol{\Xi}_k\underline{\boldsymbol{\xi}}_k\middle|\overline{\mathcal{I}}_k\right\} + S\end{aligned} \tag{29}$$

denoting the cost difference induced by the control and

$$\Delta_{est} = \sum_{i=k+1}^{K-1}\mathrm{E}\left\{\underline{\boldsymbol{\mathcal{E}}}_i^T\boldsymbol{\Xi}_i\underline{\boldsymbol{\mathcal{E}}}_i\middle|\mathcal{I}_k\right\} - \sum_{i=k+1}^{K-1}\mathrm{E}\left\{\overline{\underline{\boldsymbol{\mathcal{E}}}}_i^T\boldsymbol{\Xi}_i\overline{\underline{\boldsymbol{\mathcal{E}}}}_i\middle|\overline{\mathcal{I}}_k\right\} \tag{30}$$

identifying the cost difference induced by the estimation. While Δ_{con} can be calculated exactly, Δ_{est} cannot be computed exactly as pointed out in Schenato (2008). We will approximate Δ_{est} by the upper bound $\overline{\Delta}_{est}$ and the lower bound $\underline{\Delta}_{est}$.

With these bounds, we can redefine the decision rule (25) as

- $\Delta_{con} + \underline{\Delta}_{est} \geq 0 \rightarrow$ do not send $\underline{U}_k$,
- $\Delta_{con} + \overline{\Delta}_{est} \leq 0 \rightarrow$ send $\underline{U}_k$, (31)
- otherwise $\rightarrow$ not decidable.

In cases, where no decision can be made, we choose the controller to transmit $\underline{U}_k$ to maintain control integrity.

In the rest of this section, we derive the bounds for $\underline{\Delta}_{est}$ and $\overline{\Delta}_{est}$. Using that

$$\mathrm{E}\left\{\underline{\boldsymbol{x}}^T\mathbf{L}\underline{\boldsymbol{x}}\right\} = \mathrm{trace}\left(\mathrm{E}\left\{\mathbf{L}\right\}\mathrm{E}\left\{\underline{\boldsymbol{x}}\underline{\boldsymbol{x}}^T\right\}\right) \tag{32}$$

holds for any random matrix $\mathbf{L}$ and zero-mean random vector $\underline{\boldsymbol{x}}$ that are stochastically independent of each other and the fact that

$$\mathrm{E}\left\{\mathrm{Var}\left\{\underline{\boldsymbol{\xi}}_i\middle|\mathcal{I}_i\right\}\middle|\mathcal{I}_k\right\} = \mathrm{E}\left\{\underline{\boldsymbol{\mathcal{E}}}_i^T\underline{\boldsymbol{\mathcal{E}}}_i\middle|\mathcal{I}_k\right\} \tag{33}$$

is independent of $\underline{U}_k$ (see Fischer et al. (2013)) and is therefore equal for both cases if the controller sends $\underline{U}_k$ and if it does not, we can rewrite Δ_{est} as

$$\Delta_{est} = \sum_{i=k+1}^{k+N}\mathrm{trace}\left[\left(\mathrm{E}\left\{\boldsymbol{\Xi}_i|\,\mathcal{I}_k\right\} - \mathrm{E}\left\{\boldsymbol{\Xi}_i|\,\overline{\mathcal{I}}_k\right\}\right) \times\mathrm{E}\left\{\mathrm{Var}\left\{\underline{\boldsymbol{\xi}}_i\middle|\mathcal{I}_i\right\}\middle|\mathcal{I}_k\right\}\right] . \tag{34}$$

According to Schenato (2008), (33) is upper bounded by $\mathbf{E_0^M}$ with

$$\begin{aligned}\mathbf{E}_M^M &= \Phi_{\nu_M}\left(\mathbf{E}_M^M\right) \\ \mathbf{E}_k^M &= \Phi_{\nu_k}\left(\mathbf{E}_{k+1}^M\right) , \ k = M-1, \ldots, 0 ,\end{aligned} \tag{35}$$

where M is the length of the buffer used to store measurements received by the estimator. The term $\Phi_{\nu_h}(\mathbf{X})$ is defined by

$$\Phi_{\nu_h} = \mathbf{W} + \mathbf{AXA}^T - \nu_h\mathbf{AXC}^T\left(\mathbf{V} + \mathbf{CXC}^T\right)^{-1}\mathbf{CXA}^T , \tag{36}$$

where ν_h denotes the probability that a data packet containing the measurement $\underline{\boldsymbol{y}}_k$ is delayed by h time steps, i.e.,

$$\mathrm{P}\left(\boldsymbol{\tau}_k \leq h\right) = \nu_h .$$

The lower bound of (33) is provided by $\mathbf{D}_0^M$ with

$$\begin{aligned}\mathbf{D}_M^M &= (1-\nu_M)\left(\mathbf{Q}+\mathbf{A}\mathbf{D}_M^M\mathbf{A}^T\right)+\nu_M\mathbf{P}_\infty^e \\ \mathbf{D}_k^M &= (1-\nu_k)\left(\mathbf{Q}+\mathbf{A}\mathbf{D}_{k+1}^M\mathbf{A}^T\right)+\nu_k\mathbf{P}_\infty^e \ , \\ &\qquad k = M-1,\ldots,0 \ ,\end{aligned} \quad (37)$$

where $\mathbf{P}_\infty^e$ denotes the solution of the Riccati equation

$$\mathbf{P}_\infty^e = \Phi_1\left(\mathbf{P}_\infty^e\right) \ . \quad (38)$$

Using the fact derived in Wang et al. (1986), that for symmetric matrices $\mathbf{S}$ and $\mathbf{K}$ with $\mathbf{K} \geq 0$ it holds

$$\lambda_{min}(\mathbf{S})\text{trace}(\mathbf{K}) \leq \text{trace}(\mathbf{KS}) \leq \lambda_{max}(\mathbf{S})\text{trace}(\mathbf{K}) \ , \quad (39)$$

where $\lambda_{min}(\mathbf{S})$ and $\lambda_{max}(\mathbf{S})$ denote the smallest and the largest eigenvalue of $\mathbf{S}$, we can bound (30) by

$$\underline{\Delta}_{est} \leq \Delta_{est} \leq \overline{\Delta}_{est} \ , \quad (40)$$

with

$$\underline{\Delta}_{est} = \sum_{k+1}^{k+N} \lambda_{min}\left(\mathrm{E}\left\{\boldsymbol{\Xi}_i|\mathcal{I}_k\right\} - \mathrm{E}\left\{\boldsymbol{\Xi}_i|\overline{\mathcal{I}}_k\right\}\right)\text{trace}\left(\mathbf{D}_0^M\right) \quad (41)$$

and

$$\overline{\Delta}_{est} = \sum_{k+1}^{k+N} \lambda_{max}\left(\mathrm{E}\left\{\boldsymbol{\Xi}_i|\mathcal{I}_k\right\} - \mathrm{E}\left\{\boldsymbol{\Xi}_i|\overline{\mathcal{I}}_k\right\}\right)\text{trace}\left(\mathbf{E}_0^M\right) \ . \quad (42)$$

With (41) and (42) the decision rule (31) can be evaluated.

5. SIMULATIONS

The proposed event- and sequence-based controller is compared in simulations with the optimal sequence-based but not event-based controller presented in Fischer et al. (2013). For this purpose, we conducted a simulation with 4000 Monte-Carlo runs with 100 time steps each. The system parameters of (1) are chosen as

$$\mathbf{A} = \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} \ , \quad \mathbf{B} = \begin{bmatrix}0\\ 1\end{bmatrix} \ , \quad \mathbf{C} = [1 \ 0] \ ,$$

with initial plant state and covariances set to

$$\underline{x}_0 = \begin{bmatrix}100\\ 0\end{bmatrix} \ , \qquad \mathbf{X}_0 = \begin{bmatrix}0.5^2 & 0\\ 0 & 0.5^2\end{bmatrix} \ ,$$

$$\mathbf{W} = \begin{bmatrix}01.^2 & 0\\ 0 & 0.1^2\end{bmatrix} \ , \qquad \mathbf{V} = 0.2^2 \ .$$

The weighting matrices of the cost function (4) are

$$\mathbf{Q} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad \text{and} \quad \mathbf{R} = 1 \ .$$

Fig. 3 illustrates the probability density function of the arrival probability in the CA-channel over time delays. The probability that a data packet will suffer no delay is very low compared to probability of higher delays and there is a 10% chance of packet loss. In the SC-link, we use network A presented in Fischer et al. (2013) with a probability of 70% that the measurement will experience no delay and a negligible probability of packet loss.

We define the LQG costs per time step as LQG costs (5) normalized by the horizon length K. In Fig. 4, the LQG costs per time step of the proposed controller and the optimal sequence-based but not event-based controller derived in Fischer et al. (2013) are depicted over the control sequence length. It can be seen, that in the simulated scenario, the presented controller causes smaller LQG costs than the controller derived in Fischer et al. (2013). The costs converge to a fixed value with increasing control sequence length.

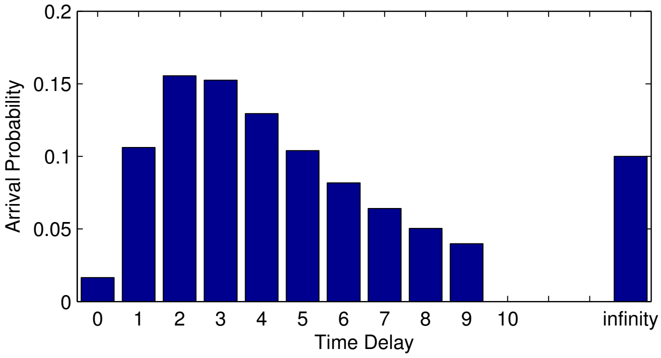

Fig. 3. Probability density function of the arrival probability of data packets over time delays.

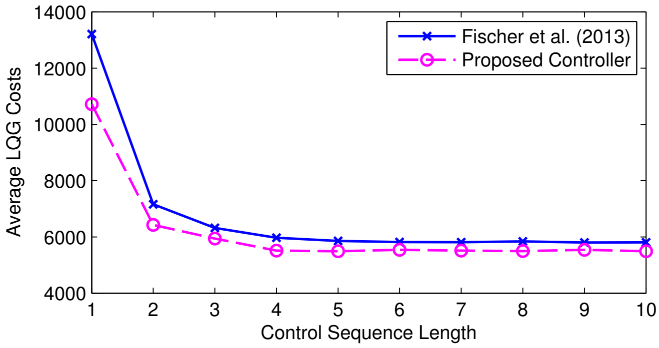

Fig. 4. Average LQG costs per time step in dependence of sequence length of the presented controller and the optimal controller derived in Fischer et al. (2013) for transmission costs $S = 5000$.

Fig. 5 shows the number of control sequence transmissions over packet length for the controller derived in Fischer et al. (2013) and the proposed controller when the transmission costs are chosen to $S = 5000$. The proposed controller conducts about 45% less control sequence transmissions than the controller from Fischer et al. (2013). The number of transmissions conducted by the proposed controller converges with increasing packet length.

6. CONCLUSION

We presented an event- and sequence-based LQG controller for NCS with a TCP-like network that is subject to packet delays and losses connecting the controller and the actuator. Concerning the sensor-controller network, no assumptions were made. The controller only transmits control sequences when sequences buffered by the actuator do not provide sufficient control quality. By doing so, we address problem (i) because the communication load in the controller-actuator channel is reduced. Packet delays and losses (problem (ii)) are mitigated by the application of the sequence-based control strategy itself. Although the proposed controller is suboptimal, it is able to maintain better performance compared to the performance of the

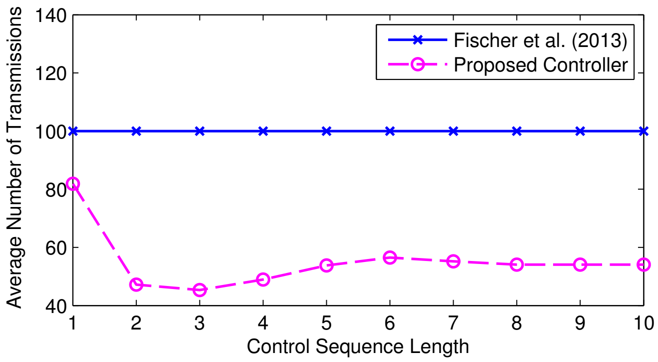

Fig. 5. Average number of control sequence transmissions performed by the presented controller and the optimal controller derived in Fischer et al. (2013). Transmission costs $S = 5000$.

optimal controller presented in Fischer et al. (2013) while conducting less control sequence transmissions.

Future work will deal with the derivation of stability criteria for the proposed controller. We also seek to find minimum transmission rate and a minimum control sequence length that guarantee stability and desired control quality for a given network. Other attempts will concentrate on finding a way to compute probability assigned to the event that the controller will transmit a future control sequence. Thus, the approximations made can be dropped and an optimal event- and sequence-based LQG controller can be derived.

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REFERENCES

Antsaklis, P. and Baillieul, J. (2007). Special Issue on Technology of Networked Control Systems. *Proceedings of the IEEE*, 95(1), 5–8.

Bemporad, A. (1998). Predictive Control of Teleoperated Constrained Systems with Unbounded Communication Delays. In *Proceedings of the 37st IEEE Conference on Decision and Control (CDC 1998)*, volume 2, 2133–2138 vol.2.

Cogill, R. (2009). Event-Based Control using Quadratic Approximate Value Functions. In *Proceedings of the 48th IEEE Conference on Decision and Control (CDC 2009)*, 5883–5888.

Fischer, J., Hekler, A., Dolgov, M., and Hanebeck, U.D. (2013). Optimal Sequence-Based LQG Control over TCP-like Networks Subject to Random Transmission Delays and Packet Losses. In *Proceedings of the 2013 American Control Conference (ACC 2013)*. Washington D. C., USA.

Fischer, J., Hekler, A., and Hanebeck, U.D. (2012). State Estimation in Networked Control Systems. In *Proceedings of the 15th International Conference on Information Fusion (Fusion 2012)*. Singapore.

Garcia, E. and Antsaklis, P. (2013). Model-Based Event-Triggered Control for Systems With Quantization and Time-Varying Network Delays. *IEEE Transactions on Automatic Control*, 58(2), 422–434.

Grune, L., Pannek, J., and Worthmann, K. (2009). A Prediction Based Control Scheme for Networked Systems with Delays and Packet Dropouts. In *Proceedings of the 48th IEEE Conference on Decision and Control (CDC 2009)*, 537–542.

Gupta, V., Sinopoli, B., Adlakha, S., Goldsmith, A., and Murray, R. (2006). Receding Horizon Networked Control. In *Proceedings of the 44th Allerton Conference on Communication, Control, and Computing*. Illinois, USA.

Heemels, W. and Donkers, M. (2013). Model-Based Periodic Event-Triggered Control for Linear Systems. *Automatica*, 49(3), 698 – 711.

Hekler, A., Fischer, J., and Hanebeck, U.D. (2012). Packet-Based Control for Networked Control Systems Based on Virtual Control Inputs. In *Proceedings of the 51st IEEE Conference on Decision and Control (CDC 2012)*. Maui, Hawaii, USA.

Hespanha, J., Naghshtabrizi, P., and Xu, Y. (2007). A Survey of Recent Results in Networked Control Systems. *Proceedings of the IEEE*, 95(1), 138–162.

Lehmann, D. and Lunze, J. (2012). Event-Based Control with Communication Delays and Packet Losses. *International Journal of Control*, 85(5), 563–577.

Lunze, J. and Lehmann, D. (2010). A State-Feedback Approach to Event-Based Control. *Automatica*, 46(1), 211 – 215.

Quevedo, D., Silva, E., and Goodwin, G. (2007). Packetized Predictive Control over Erasure Channels. In *Proceedings of the 2007 American Control Conference (ACC 2007)*, 1003–1008.

Åström, K. and Bernhardsson, B. (2002). Comparison of Riemann and Lebesgue sampling for first order stochastic systems. In *Proceedings of the 41st IEEE Conference on Decision and Control (CDC 2002)*, volume 2, 2011–2016 vol.2.

Schenato, L. (2008). Optimal Estimation in Networked Control Systems Subject to Random Delay and Packet Drop. *IEEE Transactions on Automatic Control*, 53(5), 1311–1317.

Schenato, L., Sinopoli, B., Franceschetti, M., Poolla, K., and Sastry, S. (2007). Foundations of Control and Estimation Over Lossy Networks. *Proceedings of the IEEE*, 95(1), 163–187.

Trimpe, S. and D’Andrea, R. (2012). Event-Based State Estimation with Variance-Based Triggering. In *Proceedings of the 51st IEEE Conference on Decision and Control (CDC 2012)*, 6583–6590.

Varutti, P., Kern, B., Faulwasser, T., and Findeisen, R. (2009). Event-based Model Predictive Control for Networked Control Systems. In *Proceedings of the 48th IEEE Conference on Decision and Control (CDC 2009)*, 567–572.

Wang, S.D., Kuo, T.S., and Hsu, C.F. (1986). Trace Bounds on the Solution of the Algebraic Matrix Riccati and Lyapunov Equation. *IEEE Transactions on Automatic Control*, 31(7), 654–656.

Zhang, W., Branicky, M., and Phillips, S. (2001). Stability of Networked Control Systems. *IEEE Control Systems Magazine*, 21(1), 84–99.