Improving Multitarget Tracking Using Orientation Estimates for Sorting Bulk Materials

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***Abstract*— Optical belt sorters can be used to sort a large variety of bulk materials. By the use of sophisticated algorithms, the performance of the complex machinery can be further improved. Recently, we have proposed an extension to industrial optical belt sorters that involves tracking the individual particles on the belt using an area scan camera. If the estimated behavior of the particles matches the true behavior, the reliability of the separation process can be improved. The approach relies on multitarget tracking using hard association decisions between the tracks and the measurements. In this paper, we propose to include the orientation in the assessment of the compatibility of a track and a measurement. This allows us to achieve more reliable associations, facilitating a higher accuracy of the tracking results.**

I. Introduction

The use of sensor-based sorters for sorting bulk materials allows discriminating particles based on a wide range of possible features. In particular, many particles in common bulk material sorting tasks can be classified reliably based on features that can be obtained using imaging sensors. Popular applications of sensor-based sorters include waste management, sorting industrial minerals, and ensuring quality of food. Aside from classical RGB data, the spatial distribution of the temperature or radioactivity can also be put in visual form and used for the sorting process. Not only are sensor-based sorters widely applicable, they also allow for a non-destructive and dry sorting process [1].

An example of a camera-based sorter design is illustrated in Fig. 1. Bulk material is applied to the conveyor belt and the individual particles are expected to adapt to the velocity of the belt. On the belt or after the belt, the particles are observed by a camera. At the end of the belt, each particle launches along a parabolic flight path that is altered for some particles using bursts of compressed air. The particles hit by the bursts no longer follow the parabolic flight path and land in a different container than the particles that fly unobstructed.

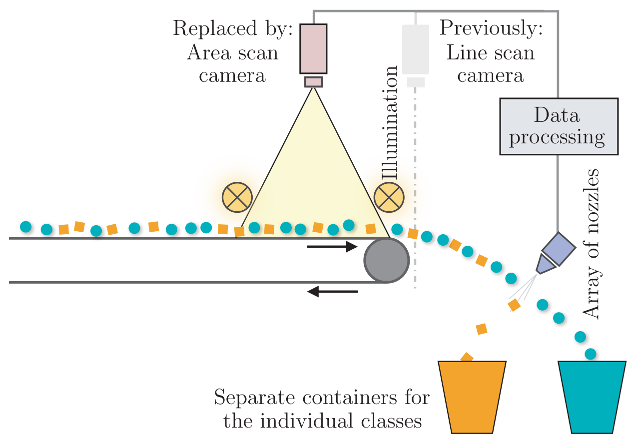

Fig. 1: Schematic view of an optical belt sorter showing the essential components. We show both the area scan camera used for the tracking-based approach and the line scan camera currently used in industrial optical belt sorters.

The delays between the observation and the separation pose a particular challenge. These delays are mainly caused by data processing times and the activation delays of the nozzles. Due to these delays and the high velocities of the particles, there is a significant displacement between the position at which the particle is observed last and the position at which it passes the separation mechanism. To bridge this gap, predictions based on motion models are required. Sensor-based sorters as currently commonly used in the industry utilize line scan cameras and assume that all particles fly straight in the transport direction of the belt. In recent work, we proposed a method to improve the sorting quality by the use of an area scan camera [2], [3]. We utilize the new data available by employing multitarget tracking to keep track of the motion of the particles traveling along the belt. Based on the tracking results, we can improve the predictions as to where the particles will pass the separation mechanism, which is why we refer to our approach as predictive tracking. Predictive tracking can not only help improve the sorting process, the observed trajectories of the particles can also be used for classification purposes [4].

The multitarget tracking algorithm employed relies on, among others, the suitability of the motion models and the accuracy of the actual measurements. Numerous multitarget tracking algorithms exist and they differ in key aspects that determine their suitability to certain applications. One important aspect is how unlabeled measurements are treated.

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If the measurements are labeled, meaning, it is known which track each measurement originated from, the multitarget tracking problem can be split up into multiple single target tracking problems, which eliminates the key challenge of the general multitarget tracking problem. For applications in which the measurements are unlabed, there are algorithms that perform hard association decisions between tracks and measurements, e.g., the global nearest neighbor [5, Sec. 6.4] (GNN), and algorithms that do not focus on a specific association decision [6]–[8]. In multi-hypothesis tracking [5, Sec. 6.7] (MHT), multiple possible hard association decisions are regarded over multiple time steps. This leads to a tree of hypotheses that is pruned as new information allowing to assess the associations more accurately comes in.

In our case, hard association decisions are useful because they allow us to accumulate information about the track contained in the image data that can later be used for classification purposes. While using multi-hypothesis tracking would be an option, real-time constraints in the sorting task [9] do not allow for this approach. Multitarget tracking approaches using hard association decisions rely on a measure of compatibility of a measurement to a track. When measuring the state, e.g., the location, directly, a distance of each track to all individual measurements is computed. Based on this distance, the GNN finds the association that minimizes the sum of the distances.

The better the individual tracks can be discriminated, the better the association and thus the tracking becomes. In [10], it is proposed to integrate additional quantities (both quantities on linear domains and on discrete domains) that are not relevant to the motion model. In our paper, we improve the accuracy of the associations in the multitarget tracking for optical belt sorters by not only keeping track of the position but also the orientation of each particle, which is a quantity on a periodic domain. We provide details how the estimation of the orientation can be used to support the association process. In our application, the benefit of using the orientation is not limited to the association problem as the separation can also be improved. For example, when regarding an oblong particle, the orientation can matter in the separation stage as the particle may or may not be hit by neighboring nozzles depending on the orientation.

In the next section, we explain the essentials of the multitarget tracking that we employ for optical belt sorters. The integration of the orientation in this framework is described in the third section. In Sec. IV, we present evaluation results. A conclusion and an outlook are provided in the last section.

II. Basics of Predictive Tracking

The processing chain to derive predictions from the obtained image data starts with image processing. Using image processing techniques, the centroids of the particles on the belt are determined. Afterward, a motion model is used to determine when and where each particle will pass the separation mechanism. For optical belt sorters using line scan cameras, the motion model is used implicitly—one or multiple of the nozzles available are activated after a certain delay from the observation of the particle. While line scan cameras can also be used to approximate the motion orthogonal to the transport direction [11], such approaches are not yet used in state-of-the-art industrial optical belt sorters.

The predictive tracking approach builds upon centroids extracted from image data of an area scan camera and treats the tracking problem as a classical multitarget tracking problem without measurement labels. While using tracking results in a higher complexity of the algorithms, more accurate predictions can be achieved as more information about the particle movement is obtained.

Let us assume for now that the number of targets measured is equal to the number of tracks in our current estimate. When using the GNN as the multitarget tracking algorithm, we require a suitable distance measure. A distance measure that is easy to employ for linear spaces such as $\mathbb{R}^n$ is the (squared) Euclidean distance. However, depending on the scenario, other distance measures may be more suitable, as becomes evident when regarding the association likelihoods [12, Sec. 10.3]. A likelihood $\ell(\hat{\underline{z}}|i)$ can be given that describes the compatibility of each track to any measurement for a single vector-valued measurement $\hat{\underline{z}}$ and the index of the track i [12, Eq. (10.22)]. If both the uncertainties in the measurements and of the tracks are normally distributed, the likelihoods can be described using normal distributions.

To derive the best association directly from the likelihoods, we can look for the association of the tracks to the measurements that maximizes the product of the individual likelihoods. As we do not allow one measurement to be associated with more than one track and vice versa, our resulting association decision can also be described by a permutation. By taking the logarithm of the product of the likelihoods and inverting the sign, we can transform the maximization problem into a problem of minimizing the sum [13, Ch. 11] of the (negative) log-likelihoods. For normal distributions, the result obtained from taking the logarithm of a single association likelihood and inverting the sign is very similar to a squared Mahalanobis distance [12, Sec. 10.3.1.2], with just an additional factor and an additive term on top. Due to this similarity, we can instead determine the association that minimizes the sum of the squared Mahalanobis distances for association problems with only normally distributed noise terms. The problem of minimizing the sum can be formulated as a classical linear assignment problem, for which fast solvers exist. In our implementation, we use the LAPJV algorithm [14], which finds the optimal solution with a complexity of $O(n^3)$. Given the association, we use one Kalman filter per particle to estimate the position and velocity of each particle.

In our scenario, particles are regularly entering and exiting the field of view. To take new particles into account, we assign each measurement a likelihood that it stems from a yet unknown track. This likelihood is high at the beginning of the observed area. While we expect most particles to appear within the range that an average particle moves during one time step (for example, $6\,\mathrm{mm}$), a hard cutoff should not be used due to the inherent variation of the velocities of the individual particles. We apply a similar scheme for the likelihood that the track is not observed again. If our prediction indicates that the track has left the observable area by the time of the next observation, we assign a high likelihood that the track is not observed. To account for missed observations and modeling uncertainties, the two likelihoods are never set to zero. To use these scenario-specific adjustments in the GNN framework, we convert the likelihoods into distances to achieve the same effect. We refer the reader to [15] for more details on the matrix generated for the use in the assignment problem solver and a more thorough discussion of the multitarget tracking as used in the predictive tracking.

III. Predictive Tracking Using the Orientation

The weakness of the GNN is that wrong associations can have a significant impact on the performance of the multitarget tracking. Furthermore, in our tracking problem, wrong associations can lead to, e.g., tracks being confused from one time step on, which can lead to significant problems in the classification. If we accumulate visual or motion-based features of the tracks, the decision may be wrong for tracks that are confused as features that actually belong to a different track are also used for the classification.

Let us say, for example, that particles A and B are particles of a food and at some point in time, the image processing has detected that a spot of particle A is covered by poisonous fungi. If the particles A and B collide toward the end of the belt, then confusing the two particles at this point may lead to particle B being separated from the stream, while particle A will fly unobstructed and become part of the final product.

Thus, good association performance is crucial to both the tracking and the classification. To improve the associations, we extend our state vector by the orientation, which can be determined using visual data for certain classes of particles. In the following, we first describe how we can keep track of the orientation while assuming the association is given and then lay out how the association can be enhanced by the use of the orientation. For simplicity, we always assume that the orientation of the particle is independent of the position of the particle and neglect any linear–circular correlations.

A. Orientation Estimation Based on Image Data

Given a non-rotationally symmetric particle, the image processing can extract a quantity representing the orientation of the particle from the image data. For example, the angle describing the direction in which the longest straight line within the object points could be used. In our scenario, we have to particularly pay attention to ambiguities in the image data. For example, cube-shaped objects appear 90-degree symmetric in the image data, while a 180-degree symmetry may be observed for other cuboids and cylinders lying sideways.

The orientation of a particle in a two-dimensional plane is a periodic quantity with an underlying circular manifold. Thus, many classical approaches to recursive Bayesian estimation can, if at all, only be applied with significant modifications and may not show the expected behavior. To correctly model probabilities on the circle, directional statistics [16], [17] can be employed. Recursive Bayesian estimators for the circle have been proposed, e.g., in [18], [19]. If the association is given, these filters allow us to estimate the orientations of the particles and model the uncertainties adequately. In the case of 90 or 180-degree symmetry, we can integrate the ambiguity using a multimodal likelihood and employ a filter that allows the modeling thereof. While filters using the Bingham distribution [20] are inherently suitable for 180-degree symmetries, flexible filters such as [21], [22] can be applied for, e.g., 120-degree or 180-degree symmetries.

Alternatively, we can define a linear mapping, e.g., from $[0, \frac{1}{2}\pi)$, $[0, \frac{2}{3}\pi)$, or $[0, \pi)$ to $[0, 2\pi)$ and use standard 2π-periodic filters for problems featuring 90, 120, or 180-degree symmetries. An easy mapping is to simply multiply all values by 2 (for 180-degree symmetry), 3 (for 120-degree symmetry) or 4 (for 90-degree symmetry). We use such a mapping to be able to use the von Mises filter [19] as implemented in [23] in our sample implementation. The estimate as provided by the von Mises filter can be transformed back via a division by the respective value to obtain a valid estimate in the limited distinguishable range of angles.

B. Using the Orientations for the Association

To improve our multitarget tracking, we now use the orientations in the association process. As we assume the position component and the orientation component to be independent, we can split the likelihood of association of an individual measurement up into a part concerning the position and a part concerning the orientation. The likelihood of association of the measurement $\hat{\underline{z}}$ to the ith track is given by

$$\ell(\hat{\underline{z}}|i) = \int_{\mathbb{R}^2 \times [0,2\pi)} \ell(\hat{\underline{z}}, \underline{x}|i)\, d\underline{x} \ .$$

We now call the part of the state $\underline{x}$ describing the position in the two-dimensional plane $\underline{x}^{\text{pos}}$ and the (scalar) part describing the orientation x^{ang}. For a measurement $\hat{\underline{z}}$ comprising information about the position $\hat{\underline{z}}^{\text{pos}}$ the orientation $\hat{z}^{\text{ang}}$, we can rewrite the likelihood of association as

$$\ell(\hat{\underline{z}}|i) = \int_{\mathbb{R}^2 \times [0,2\pi)} \ell(\hat{\underline{z}}^{\text{pos}}, \hat{z}^{\text{ang}}, \underline{x}^{\text{pos}}, x^{\text{ang}}|i)\, d\underline{x} \ .$$

Due to our assumption that the position and the orientation are independent, we can write the likelihood as

$$\begin{aligned} \ell(\hat{\underline{z}}|i) &= \int_{\mathbb{R}^2 \times [0,2\pi)} \ell(\hat{\underline{z}}^{\text{pos}}, \underline{x}^{\text{pos}}|i)\, \ell(\hat{z}^{\text{ang}}, x^{\text{ang}}|i)\, d\underline{x} \\ &= \underbrace{\int_{\mathbb{R}^2} \ell(\hat{\underline{z}}^{\text{pos}}, \underline{x}^{\text{pos}}|i)\, d\underline{x}^{\text{pos}}}_{=:\ell(\hat{\underline{z}}^{\text{pos}}|i)} \underbrace{\int_{[0,2\pi)} \ell(\hat{z}^{\text{ang}}, x^{\text{ang}}|i)\, dx^{\text{ang}}}_{=:\ell(\hat{z}^{\text{ang}}|i)} \ . \end{aligned}$$

As previously stated, the problem of finding the association that maximizes the product of the likelihoods can be reformulated as a problem of minimizing the sum of the negative log-likelihoods. If we apply the logarithm and invert the sign, we can see from

$$-\log \ell(\hat{\underline{z}}|i) = -\log \ell(\hat{\underline{z}}^{\text{pos}}|i) - \log \ell(\hat{z}^{\text{ang}}|i)$$

that we can regard the two parts separately and combine them afterward.

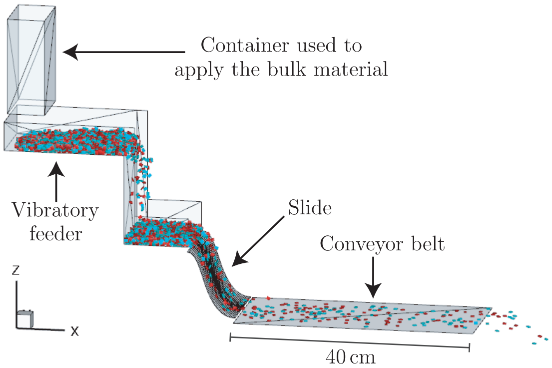


Fig. 2: Three-dimensional model of the optical belt sorter as used in the DEM simulation.

Assuming the uncertainties in the estimated positions and the measurements are normally distributed, we can follow the scheme as given in [12, Eq. (10.22)–(10.24)] to obtain a formula for the likelihood $\ell(\hat{\underline{z}}^{\mathrm{pos}}|i)$ that only requires evaluating the density of a normal distribution. Further, we can reduce $-\log \ell(\hat{\underline{z}}^{\mathrm{pos}}|i)$ to a squared Mahalanobis distance [12, Eq. (10.25)–(10.29)] between the measurement $\hat{\underline{z}}^{\mathrm{pos}}$ and the position estimate $\underline{x}_i^{\mathrm{pos}}$ of the ith track. However, the formula for $-\log \ell(\hat{z}^{\mathrm{ang}}|i)$ does not reduce to an intuitive and cheap to calculate distance even if the uncertainties in the estimated and measured angles are von Mises distributed. Therefore, we replace $-\log \ell(\hat{z}^{\mathrm{ang}}|i)$ with a common distance measure on the circle [17, Eq. (1.3.7)] called the cosine distance

$$d(\hat{z}^{\mathrm{ang}}, \hat{x}_i^{\mathrm{ang}}) = 1 - \cos(\hat{z}^{\mathrm{ang}} - \hat{x}_i^{\mathrm{ang}})$$

between the measurement $\hat{z}^{\mathrm{ang}}$ and the estimate of the ith track $\hat{x}_i^{\mathrm{ang}}$. The squared Mahalanobis distance for the position is then combined with the cosine distance for the orientation via a weighted summation. This combination induces that the distance-like penalty terms representing the likelihood that the measurement belongs to a yet unknown track and the likelihood that the track is not in sight have to be adjusted when integrating the orientation in the association problem.

Unlike the squared Mahalanobis distance, the cosine distance does not take the uncertainties in the estimates and measurements into account. Currently, we combine the squared Mahalanobis distance with the cosine distance using a fixed weighting factor. In future work, we plan to use a weighting or a distance measure that respects the uncertainty in the orientation estimate and the orientation measurement.

IV. Evaluation

To evaluate our proposed extension of the multitarget tracking algorithm, we regard the association decisions performed when including or excluding the angle in the association process. For this, we simulate feeding a batch of particles to TableSort, a small experimental optical belt sorter that we use in our project. The simulations of TableSort are based on the discrete element method [24] (DEM) and are explained in more detail in [3], [25]. The three-dimensional model of the sorter used in the simulation is depicted in Fig. 2. A realistic belt speed of $1.15\,\mathrm{m/s}$ was chosen for the simulation. As the bulk material for our evaluation, we use a batch of 4113 wooden cuboids. The models are based on real cuboids as shown in Fig. 3. The parameters of the cuboids that are relevant to the simulation were determined experimentally according to procedures described in [26].

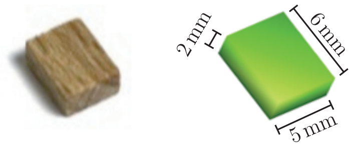


Fig. 3: The real cuboid that the stimulated particles are based on is shown on the left and the modeled cuboid on the right.

A. Evaluation Criteria

Unlike real data, the data obtained from the numerical simulation provide us with the entire trajectories of the individual particles. Using the ground truth trajectories, we can generate measurements with known labels. These labels are then used to resolve which track gave rise to which measurement. This information is solely used for validation purposes and is not made available to the multitarget tracking algorithm. While we know the entire trajectories of the particles including their positions during the feeding process, only measurements of particles on the belt are passed on to the multitarget tracking algorithm. Although our ground truth has a higher temporal resolution, we only provide measurements to the multitarget tracking at $200\,\mathrm{Hz}$, which is a realistic frame rate for area scan cameras in real world applications.

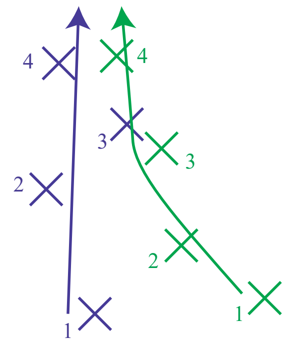
Our algorithm saves the indices of all measurements that are assigned to each track. Using the information saved during the generation of the measurements, we use these indices to look up the IDs of the ground truth tracks to which the measurements belong. Based on the list of IDs of the tracks whose measurements were assigned to an estimated track, we define two types of errors.

If the list of a single estimated track contains IDs of multiple ground truth tracks, this is proof of an incorrect association. In the following, we refer to this as an error of the first kind. This measure of error does not respect errors occurring when a measurement is erroneously deemed to stem from a new track although the corresponding track has already been observed. An extreme example is a scenario in which the multitarget tracking erroneously does not assign any measurement to existing tracks. In this case, all measurements would be assigned to new tracks in each time step, which would not result in errors of the first kind.

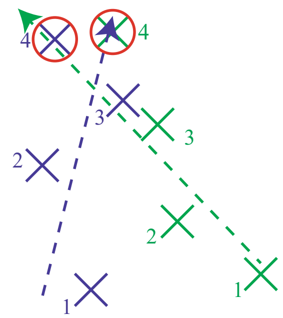
To also respect these false associations, we define a second measure of error. If measurements of a single ground truth track were used in multiple estimated tracks, this counts as an error of the second kind. As illustrated in Fig. 4, these two types of errors are not mutually exclusive. If, e.g., two tracks are confused from a certain time step on, this will result in both types of errors.

B. Evaluation Results

In all scenarios regarded, we keep track of the position of each particle in the two-dimensional plane in which the belt lies and use the identity matrix as the measurement matrix for the position measurements. We use a constant velocity model



(a) True trajectory and the measurements generated.



(b) Trajectories estimated using the measurements. The errors can be observed in the last time step.

Fig. 4: Illustration of how two tracks can be confused, which leads to both kinds of errors. Measurements of different ground truth tracks were used in both tracks and the two tracks share measurements of a common ground truth track. The measurements are shown as crosses with the corresponding time indices and erroneous associations are circled in red. Ground truth tracks are shown using solid lines, whereas dashed lines are used for estimated tracks.

as the system model for the position component of the state. As the measurement model for estimating the orientation, we use an identity model with additive noise that takes the periodicity into account. A random walk model adapted to the periodic case is used as the system model for the orientation component.

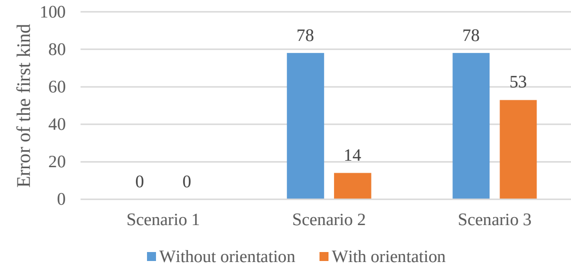
In our first scenario, we evaluate the performance of the multitarget tracking when the ground truth data are used directly as the measurements. As shown in Fig. 5, this results in zero errors of both kinds, regardless whether the orientation is used or not. Thus, when the measurements are highly accurate and the system model is suitable, using the orientation may not be required to reliably determine the correct associations.

In real world scenarios, the position measurements always contain some kind of measurement noise. To account for such noises in our second scenario, we add an additive noise term

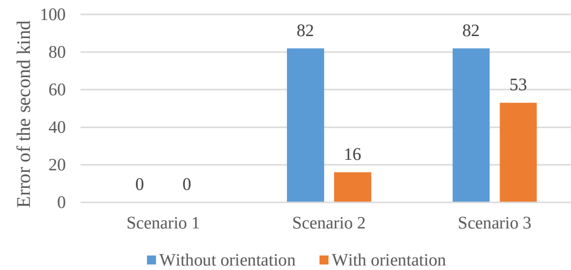
$$\underline{v}^{\text{pos}} \sim \mathcal{N}(0\,\text{m}, 0.001\,\text{m}^2 \cdot \mathbf{I}) \ ,$$

with the identity matrix $\mathbf{I}$ to each position measurement. With $\mathbf{C} = 0.001\,\text{m}^2 \cdot \mathbf{I}$, the standard deviation is about $0.032\,\text{m}$ along each of the two axis. As the distance an average particles travels in one time step is approximately $0.0055\,\text{m}$ in the simulation, this is clearly a non-negligible noise term. We did not use our knowledge about the precise parameters of the noise when choosing the parameters of the multitarget tracking algorithm as such knowledge is usually unavailable in real world applications.

The ground truth data obtained using the DEM simulation include a full rotation matrix for each particle in every time step. Since only the angle around the axis that is a normal to the plane in which the belt lies can be reliably obtained from the two-dimensional image data captured from above, we reduce our rotation matrix to that single angle. For this, we can first multiply the rotation matrix with the vector



(a) Errors of the first kind for the scenarios evaluated.



(b) Errors of the second kind for the scenarios evaluated.

Fig. 5: Visualization of the evaluation results for both approaches and all three scenarios.

$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$. Then, the third entry is discarded to perform the projection on the two-dimensional plane. The angle on the plane can then be calculated using the $\operatorname{atan2}$. These steps can be efficiently combined in one step. For a rotation matrix $\mathbf{R} \in \mathbb{R}^{3\times 3}$, we can use $\operatorname{atan2}(r_{2,1}, r_{1,1})$ to calculate the desired angle.

In the second scenario, we did not add any noise to the orientation. Further, the ambiguity in the orientation stemming from the symmetries of the particles was disregarded. This was possible as the rotation matrices obtained from the simulation do not suffer from such ambiguities. As evident from Fig. 5, using the orientation significantly reduces the errors in the associations in this scenario.

In the third scenario, we add a von Mises distributed noise term of $\boldsymbol{v}^{\text{ang}} \sim \mathcal{VM}(0, 100)$ to all angle measurements to account for noise in the orientation measurements in real world scenarios. Further, we account for the symmetries of the particles. While the thin cuboids as shown in Fig. 3 are, strictly speaking, only 180-degree symmetric, we say that in the worst case, the image processing cannot reliably determine the longer side. Therefore, we assume the particle to be 90-degree symmetric, which is even more challenging than a 180-degree symmetry. To generate the measurements, we first add $\boldsymbol{v}^{\text{ang}}$ to the ground truth angles. Then, we multiply all angles by 4 and take them mod 2π, which allows us to use the 2π-periodic von Mises filter and erases the information about the precise orientation derived from the ground truth data. The 90-degree symmetry worsens the effect of $\boldsymbol{v}^{\text{ang}}$ added to the angles as it limits the range of usable angles from $[0, 2\pi)$ to $[0, \frac{1}{2}\pi)$. As shown in Fig. 5, the uncertainties and symmetries reduce the benefit of using the orientation. Nonetheless, an improvement can be observed even in this challenging scenario.

V. CONCLUSION

In this paper, we have proposed a novel extension to the predictive tracking scheme used for sorting bulk materials using optical belt sorters. Using an approach based on directional statistics, we are able to estimate the orientation of the particles. This information is not only useful by itself, e.g., for motion-based classification or to improve the separation using the nozzles, it also allows improving the associations of the multitarget tracking scheme. Accurate associations are important both for precise tracking results and for being able to accumulate visual features of the particles to guide the separation decision.

The enhancement described is a purely software-based improvement. For n active tracks, the distance calculation for the association problem is in $O(n^2)$ if the number of measurements and tracks is approximately equal. Given the association decision, the additional overhead to estimate the orientation is in $O(n)$. Since the assignment problem solver has a run time in $O(n^3)$, the asymptotic complexity of the algorithm is not changed. To use this extension in real world applications, a fast image processing routine to extract the orientations of the particles is also required.

Future work will include investigating other distance measures for the orientation term or combining the position and orientation components in a different manner. Further, we plan to use the orientation estimates in the classification. Additionally, the sizes of the particles could also be estimated and used for the association decision. Moreover, using other system models may enhance the estimation of the position and orientation.

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