A Model-Based Framework for Optimal Measurements in Machine Tool Calibration

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***Abstract*— Calibration is the procedure of quantifying mechanical deficiencies of machines and compensating them by appropriate adjustment. This paper introduces a model-based measurement framework for improving calibration procedures of machine tools. The goal is to precisely estimate the mechanical deficiencies based on a minimal number of measurements. For that purpose, the mechanical deficiencies of linear and rotary joints are modeled using splines. Uncertainties of the deficiency model are formulated stochastically, which allows incorporation of imprecise measurement data and prediction of optimal measurement parameters. We derive a method for optimally estimating a set of splines, i.e., joint errors, based on a set of measurements and for predicting the optimal joint configuration for new measurements.**

***Index Terms*— calibration, machine tool, stochastic splines**

I. Introduction

Demands on the mechanical precision of machine tools and robots in manufacturing are becoming higher and higher. Tolerances within μm-range are standard today. To ensure such precision, a calibration needs to be done in regular time intervals. It consists of the measurement of deviations caused by mechanical deficiencies and wear on one hand. On the other hand, it includes the adjustment of the machine to correct these deviations. We only deal with the measurement procedure here. Calibration techniques are well understood [1] and we found only few recent works on that topic [2], [3]. Nevertheless, all problems concerning calibration have not been solved satisfactorily: A calibration procedure of a modern machine tool takes several days and has to be performed manually by an expert. The required time increases exponentially with the number of machine joints. Since manufacturing has to be stopped during that procedure, any means helping to reduce that time needed for calibration significantly lowers the costs of operation.

We propose a model-based framework for determining optimal measurement positions for a calibration procedure of a machine tool, which minimizes the number of required measurements. The mechanical deficiencies will be modeled using splines with a stochastic treatment of uncertainties. It will be shown that linear models are sufficient for coping with a large class of the machine tools being in use today.

The paper is organized as follows: We begin with describing the problem of model-based measurement and optimal measurement prediction. In section III, we show that cubic splines are an appropriate model for linear and rotary joints of machine tools. We also show how to incorporate stochastic uncertainties into splines and how to reduce these through measurements. We finish that section by deriving optimal measurement positions for splines. In the next section, we introduce an algorithm for calibrating a machine with multiple joints, i.e., for determining parameters of multiple splines based on measurements. The performance of the proposed new approach is demonstrated by means of simulations. We finish the paper with an outlook to future work.

Fig. 1. The DMU 60 machine tool from Deckel-Maho with five joints.

II. Problem Formulation

Many machine tools consist of two or three orthogonal linear joints and one or two orthogonal rotary joints. As a result, the forward kinematics become quite simple and can be written for a robot or machine with q degrees of freedom as

$$\underline{x}_{TCP} = \underline{K}_1(t^1) + \underline{V}_1(t^1) + \ldots + \underline{K}_q(t^q) + \underline{V}_q(t^q) \ , \quad (1)$$

where $\underline{t} = [t^1, t^2, \ldots t^q]^T$ describes the joint configuration of the machine tool. The world coordinate system is placed in the table holding the workpiece and $\underline{x}_{TCP}$ is the position of the tool center point (TCP). The TCP is also the point where we conduct the measurements.

$\underline{K}_i(t^i), i = 1\ldots,q$, are functions describing the joint motion depending on the joint pose parameter t^i. $\underline{V}_i(t^i)$ summarizes the mechanical deficiencies for the i^{th} joint. Nonlinearities are often avoided in machine tools, because it keeps the kinematics simple and robust. We will only discuss the translational components of the TCP, since the rotational ones play a minor role in machine tools.

The objective of this paper is to determine all $\underline{V}_i(t^i)$ with a minimal number of scalar measurements $y_i, i = 1,\ldots,m$, which are modeled by a measurement equation

$$y_i = \underline{h}_i^T \, \underline{x}_{i,TCP} + v_i \ ,$$

where the vector $\underline{h}_i^T$ maps i^{th} TCP-Position $\underline{x}_{i,TCP}$ onto the i^{th} measurement y_i and v_i describes the measurement noise. Since the machine configurations $\underline{t}_i$ strongly influences the measurement y_i and thus the quality of the estimate, an optimal machine configurations $\underline{t}_i$ has to be determined for every measurement.

To achieve this, the following measurement procedure is utilized: We begin with an initial estimate for the deficiencies described by the expected value $\hat{\underline{A}}^0$ and the covariance matrix $\mathbf{C}^0$, which are explained in IV. A weighting function $J(\mathbf{C}^0)$ quantifies the quality of the estimate. If a measurement y_1 is received, the estimate can be improved by calculating $\hat{\underline{A}}^1$ and $\mathbf{C}^1$ using stochastic combination. To obtain an optimal machine configuration $\underline{t}_1$ for the next measurement, the weighting function $J(\mathbf{C}^1(\underline{t}))$, which depends on the machine configuration $\underline{t}$ through $\mathbf{C}^1(\underline{t})$, is minimized. The argument of the minimum $\underline{t}_1$ is the optimal machine configuration for the next measurement. After the measurement is carried out, the next optimal machine configuration is calculated.

In this paper we show that this problem can be solved by finding parameters of cubic splines.

III. Modeling with Splines

Splines are frequently used for interpolating, especially linear, statistical data [4]. Incorporating Bayesian models has been reported in [5], [6]. These attempts lack an appropriate stochastic formulation for the use in measurement tasks. Here we use splines not for mere interpolation purposes, but as a physical model. This allows us to formulate modeling and measurement uncertainties. For this application, it is sufficient to use cubic splines, but it is easily possible to apply this procedure to other kinds of splines.

A. Cubic Splines

A cubic spline $S(t)$ is a continuous piecewise defined function defined by 3^{rd} order polynomials

$$S_i(t) = a_i + b_i(t-t_i) + c_i(t-t_i)^2 + d_i(t-t_i)^3 \ , \quad (2)$$

$t \in [t_i, t_{i+1}]$, $i = 1,\ldots,n-1$, over the interval $[t_1,t_n]$, of which the first and second deviation are continuous as well. If $d^2S(t_1)/dt^2 = d^2S(t_n)/dt^2 = 0$, we have so-called natural splines.

If the coefficients $[a_1 \ldots a_n]$ and $[t_1 \ldots t_n]$ are given, the remaining coefficients can be calculated for natural splines as

$$\begin{aligned} a_i &= S(t_i) \ , & (3)\\ b_i &= \frac{a_{i+1}-a_i}{h_i} - \frac{c_{i+1}+2c_i}{3} h_i \ , & (4)\\ c_i &= -\frac{h_{i-2}}{h_{i-1}} c_{i-2} - 2\left(\frac{h_{i-2}}{h_{i-1}}+1\right) c_{i-1} + \\ &\quad 3\left(\frac{a_i-a_{i-1}}{h_{i-1}^2} - \frac{a_{i-1}-a_{i-2}}{h_{i-1}h_{i-2}}\right) \ , & (5)\\ d_i &= \frac{c_{i+1}-c_i}{3h_i} \ , & (6) \end{aligned}$$

with $i = 1\ldots n-1$ and $h_i = t_{i+1} - t_i$ [7]. For that reason $[t_i, a_i]$ are called the control points of the spline.

If $S(t_1) = S(t_n)$, $dS(t_1)/dt = dS(t_n)/dt$ and $d^2S(t_1)/dt^2 = d^2S(t_n)/dt^2$, we speak of cyclic splines. The coefficients for the first interval of a cyclic spline are then given by

$$\begin{aligned} a_1 &= S(t_1) = S(t_n) = a_n & (7)\\ b_1 &= \frac{a_2-a_1}{h_1} - \frac{c_2+2c_1}{3} h_2 \ , & (8)\\ c_1 &= -\frac{h_{n-1}}{h_n} c_{n-1} - 2\left(\frac{h_{n-1}}{h_n}+1\right) c_n + \\ &\quad 3\left(\frac{a_1-a_n}{h_n^2} - \frac{a_n-a_{n-1}}{h_n h_{n-1}}\right) \ , & (9)\\ d_1 &= \frac{c_2-c_1}{3h_1} \ . & (10) \end{aligned}$$

The coefficients for the second and the last interval can be derived analogously by applying the control points $[t_{n-1}, a_{n-1}]$, $[t_n, a_n]$, $[t_2, a_2]$, and $[t_3, a_3]$ to (3) – (6).

Note that all a_i enter $S(t)$ linearly, so (4) – (10) can be rewritten in vector matrix notation as

$$\underline{b} = \mathbf{M}_1\underline{a}, \ \ \underline{c} = \mathbf{M}_2\underline{a}, \ \ \underline{d} = \mathbf{M}_3\underline{a} \ , \quad (11)$$

where $\mathbf{M}_{1,2,3} = \mathbf{M}_{1,2,3}(t_1,\ldots,t_n)$. The coefficient vectors are defined as $\underline{a} = [a_1 \ldots a_n]^T$, $\underline{b} = [b_1 \ldots b_{n-1}]^T$, $\underline{c} = [c_1 \ldots c_{n-1}]^T$ and $\underline{d} = [d_1 \ldots d_{n-1}]^T$. Note that Splines have the property that for a set of given points $[t_1, a_1] \ldots [t_n, a_n]$, the resulting function $S(t)$ minimizes the total curvature

$$\int_{t_1}^{t_n} \left(\frac{d^2}{dt^2} S(t)\right)^2 dt \rightarrow \min \ . \quad (12)$$

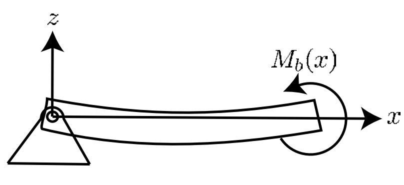

Fig. 2. A beam deflected in z-direction which is fixed on the left end.

B. Modeling of the Deflection in Linear Joints

Deficiencies will be modeled as an elastic beam, as shown in figure 2. The deflection is caused by external forces and moments at fixed (control) points. At each point x in the beam, a moment $M_b(x)$ exists, which is caused by external forces and moments. It is compensated by an elastic deflection

$$M_b(x) = -B_y \frac{d^2 y(x)}{dx^2} \ , \tag{13}$$

where B_y is a constant flexural rigidity [8]. Solving this linear differential equation gives

$$y(x) = g_R(x) + a + bx + cx^2 + dx^3 \ , \tag{14}$$

where $g_R(x)$ contains the boundary conditions consisting of moments and forces at control points x_i. Equation (14) shows that an elastic beam, which is fixed at discrete points, can be modeled as a cubic spline. We utilize these relations as basis for modeling error in linear joints: At each control point x_i, a virtual force or a virtual moment bends the beam away from the perfect straight line. If it is assumed that the spline stays within a tolerance interval $S(x) \in [S_u, S_l] \ \forall x \in [x_1, x_n]$ and it has the maximum curvature k_{max}, it can be shown that the maximum distance x_d between two control points can be approximated by

$$x_d \leq \sqrt{\frac{6(S_u - S_l)}{k_{max}}} \ . \tag{15}$$

For linear joints the pose parameter t resembles the position x on the beam. How to determine the values of the coefficients by measurements is shown in section III-D.

C. Modeling Errors in Rotary Joints

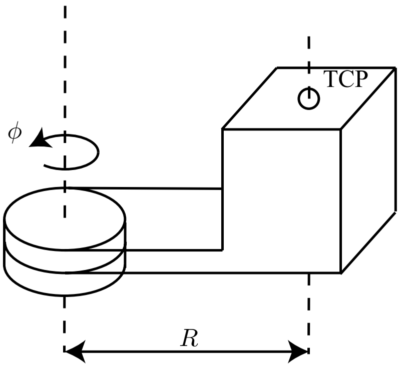

Fig. 3. Scheme of a rotary joint.

To model a rotary joint we use the fact that most machine tools have orthogonal rotary joints. Figure 3 shows a schematical rotary joint with its rotation axis on the z-axis. The TCP turns the rotation axis in a distance of the radius R. The translational error $\underline{V}_z(\phi)$ in z-direction is modeled as a cyclic spline with the joint angle ϕ as its parameter. To model a tilt between the real axis and the ideal rotation axis, the spline should have at least two control points. Using more control points allows to model higher order precessions. The joint pose parameter t resembles the angle ϕ.

D. Incorporating Uncertainties

From now on, we assume given control points $t_1 \dots t_n$. The vector $\underline{\mathbf{a}} = [\mathbf{a}_1 \dots \mathbf{a}_n]^T$ of the control point values is now interpreted as a stochastic variable. We assume its distribution to be an n-dimensional Gaussian density

$$f_a(\underline{a}) = \frac{\exp\left\{-\frac{1}{2}(\underline{a} - \underline{\hat{a}})^T \mathbf{C}_a^{-1} (\underline{a} - \underline{\hat{a}})\right\}}{\sqrt{(2\pi)^n \det(\mathbf{C}_a)}} \tag{16}$$

with expected value $\underline{\hat{a}}$ and covariance-matrix $\mathbf{C}_a$. The expected values of the other coefficients $\underline{\hat{b}}$, $\underline{\hat{c}}$ and $\underline{\hat{d}}$ can be calculated with (11), the covariance matrices

$$\mathbf{C}_b = \mathbf{M}_1 \mathbf{C}_a \mathbf{M}_1^T, \ \ \mathbf{C}_c = \mathbf{M}_2 \mathbf{C}_a \mathbf{M}_2^T, \ \ \mathbf{C}_d = \mathbf{M}_3 \mathbf{C}_a \mathbf{M}_3^T \tag{17}$$

can be calculated from $\mathbf{C}_a$. The expected value of the position $t_p \in [t_i, t_{i+1}]$ is

$$\hat{S}(\bar{t}_p) = (\underline{m}_p)^T \, \underline{\hat{a}} \tag{18}$$

with

$$(\underline{m}_p)^T = \underline{\delta}_i + (\underline{m}_1^i)^T (\bar{t}_p) + (\underline{m}_2^i)^T (\bar{t}_p)^2 + (\underline{m}_3^i)^T (\bar{t}_p)^3 \tag{19}$$

and $\bar{t}_p = t_p - t_i$. $(\underline{m}_{1,2,3}^i)^T$ is the i-th row vector of $\mathbf{M}_{1,2,3}$. $\underline{\delta}_i = [\delta_{1i} \dots \delta_{ni}]^T$ is vector of Kronecker-Symbols selecting the i-th value of $\underline{\hat{a}}$. The corresponding variance is given by

$$\sigma_{t_p}^2 = \underline{m}_p^T \mathbf{C}_a \underline{m}_p \ . \tag{20}$$

Having the expected value and the variance for every point on the spline, the density function is derived.

For a given measurement $\hat{y}_p = S(x_p) + v$ at the position x_p with variance σ_v^2, we want to improve the estimate on the control points $\underline{\mathbf{a}}$. Since we only have Gaussian distributions and linear relations, we can use linear stochastic combination [9]. The improved expected value $\underline{\hat{a}}^e$ and the covariance matrix $\mathbf{C}_a^e$ become

$$\underline{\hat{a}}^e = \left(\mathbf{I} - \underline{k}_p \, \underline{m}_p^T\right) \underline{\mathbf{a}} + \underline{k}_p \, \hat{y}_p \tag{21}$$

$$\mathbf{C}_a^e = \left(\mathbf{I} - \underline{k}_p \, \underline{m}_p^T\right) \mathbf{C}_a \left(\mathbf{I} - \underline{k}_p \, \underline{m}_p^T\right)^T + \underline{k}_p \sigma_v^2 \underline{k}_p^T \tag{22}$$

with

$$\underline{k}_p = \frac{\mathbf{C}_a \underline{m}_p}{\sigma_v^2 + \underline{m}_p^T \mathbf{C}_a \underline{m}_p} \ . \tag{23}$$

E. Predicting the Optimal Measurement Position

We want to obtain an optimal measurement position t_p, which maximally decreases the variance σ_t^2 for all spline points. To derive the optimal position, we begin by defining an interval variance

$$\sigma_i^2 = \int_{t_i}^{t_{i+1}} \mathrm{Cov}\{S(t)\} dt \tag{24}$$

$$= \int_{t_i}^{t_{i+1}} \mathrm{Cov}\{(\underline{m}_t^i)^T \underline{\mathbf{a}}\} dt$$

$$= \int_{t_i}^{t_{i+1}} (\underline{m}_t^i)^T \mathbf{C}_a \underline{m}_t^i dt \tag{25}$$

for $t \in [t_i, t_{i+1}], i = 1, \dots, n$. Since in (25) t occurs only in the deterministic m_p^i, the integration can be done independently of $\mathbf{C}_a$. It can be shown that the integral in

(25) can be eliminated by integration by parts, since the fourth derivative of $\underline{m}_t^i$ is zero.

The weighting function

$$J(t) = \sum_{i=1}^{n} \sigma_i^2 \rightarrow \min \quad (26)$$

is the sum of the interval variances σ_i^2, quantifying the quality of the estimate. Since $J(t)$ only depends on $\mathbf{C}_a$, which can be calculated using (22) without knowing the measurement values $\hat{y}_p$, $J^e(t)$ can be calculated for subsequent measurements, using $\mathbf{C}_a^e$ and $(\sigma_i^e)^2$ from (22) – (26).

For finding the minimum, we calculate the real roots $t_j^r, j = 1, \ldots, r$ of the first derivative of $J(x)$ and pick the minimal value from

$$t_p = \arg\left(\min_t \{J(t_1), J(t_1^r), \ldots J(t_r^r), J(t_n)\}\right) \ , \quad (27)$$

with the argument $t \in [t_1, t_1^r, \ldots, t_r^r, t_n]$ being the roots t_i^r and the interval borders t_j. For the sake of brevity, we will not show that the calculation of the roots boils down to finding the roots of a tenth-order polynomial for each interval.

IV. Modeling and Calibrating a Machine

With results from the previous section, it is now possible to formulate a calibration procedure for a given machine tool with q joints:

1) First the forward kinematics have to be derived from the machine geometry. Then the calibration parameters have to be defined. Offsets of linear joints either go into the calibration parameter of an orthogonal joint or can be modeled as a one control point spline. The number of control points for linear joints can be approximated using (15). For rotary joints, empirical information on the precession has to be employed. That information has to be brought in a form analogous to (1) as

$$\underline{x}_{TCP} = \underline{K}(t_1, \ldots, t_q) + \underline{S}_1(t_1) + \ldots + \underline{S}_q(t_q) \ , \quad (28)$$

where $\underline{S}_1(t_1), \ldots, \underline{S}_q(t_q)$ are vectorial splines modeling the calibration parameters $\underline{\mathbf{a}}_1 \ldots \underline{\mathbf{a}}_q$ with the expected value $\underline{\hat{a}}_1 \ldots \underline{\hat{a}}_q$ and the covariance matrices $\mathbf{C}_{a1} \ldots \mathbf{C}_{aq}$. $\underline{x}_{TCP}$ is assumed to be a p-dimensional vector.

2) To incorporate stochastic information, the measurement $\mathbf{y}$ should be Gaussian as well having the expected value $\hat{y}$ and the variance σ_v^2 of the measurement noise v. Then the measurement equation

$$y = \underline{h}^T \underline{x}_{TCP} + v \quad (29)$$

$$= \underline{\bar{h}}^T \begin{bmatrix} \underline{a}_1 \\ \vdots \\ \underline{a}_q \end{bmatrix} + v = \underline{\bar{h}}^T \underline{A} + v \quad (30)$$

has to be determined, which we assume to be linear. All coefficients are collected in one large vector and $\underline{\bar{h}}^T$ can be calculated using (3) – (11), (18) and (20).

3) Using stochastic combination analogously to (21) – (23), an improved expected value and covariance

$$\underline{\hat{A}}^e = \left(\mathbf{I} - \mathbf{K}\underline{\bar{h}}^T\right) \underline{\hat{A}} + \mathbf{K}\underline{\hat{y}} \quad (31)$$

$$\mathbf{C}_A^e = \left(\mathbf{I} - \mathbf{K}\underline{\bar{h}}^T\right) \mathbf{C}_A \left(\mathbf{I} - \mathbf{K}\underline{\bar{h}}^T\right)^T + \mathbf{K}\mathbf{C}_v\mathbf{K}^T \quad (32)$$

with

$$\mathbf{K} = \mathbf{C}_A\underline{\bar{h}}\left(\mathbf{C}_v + \underline{\bar{h}}^T\mathbf{C}_A\underline{\bar{h}}\right)^{-1} \quad (33)$$

can be determined.

4) For predicting the optimal measurement position, we proceed analogously to (24) – (27). Assuming that each joint $t^k, k = 1, \ldots, q$ has a p-dimensional vectorial Spline with n control points, the interval variances and covariances for the corresponding splines with $r, s = 1, \ldots, n$ become

$$(\sigma_{i,rs}^k)^2 = \int_{t_i^k}^{t_{i+1}^k} (\underline{m}_{t^k}^r)^T \mathbf{C}_a \underline{m}_{t^k}^s \, dt^k \ , \quad (34)$$

which are used to define an interval covariance matrix

$$\mathbf{C}_{i,k}^I = \begin{bmatrix} (\sigma_{i,11}^k)^2 & \ldots & (\sigma_{i,1p}^k)^2 \\ \vdots & & \vdots \\ (\sigma_{i,p1}^k)^2 & \ldots & (\sigma_{i,pp}^k)^2 \end{bmatrix} \ . \quad (35)$$

The weighting function results in

$$J = \sum_{k=1}^{q} \sum_{i=1}^{n} \text{trace}(\mathbf{C}_{i,k}^I) \ . \quad (36)$$

For finding the roots, still only tenth-order polynomials have to be solved for each interval. The number of polynomials increases exponentially with the number of joints and linearly with the number of control points.

5) Having determined the weighting function (36), the actual measurement procedure can begin: Starting off with an expected value $\underline{\hat{A}}^0$ and covariance matrix $\mathbf{C}_A^0$, the optimal machine configuration is determined by

$$\underline{t}_p = \arg\left(\min_{\underline{t}} \{J^1(\underline{t})\}\right) \ . \quad (37)$$

Then a measurement y_p at the position $\underline{t}_p$ is carried out and the estimate of the covariance parameters is enhanced using stochastic combination according to (31) – (33). This procedure is repeated until the m^{th} weighting function $J^m(t)$ is sufficiently small. For simplicity reasons, only one configuration for the next measurement is calculated here. With the same approach m configurations can be calculated in advance as well. It can be shown that the order, in which the m measurements are carried out, does not matter for scalar measurements y.

These five steps are needed to accomplish an optimal measurement procedure.

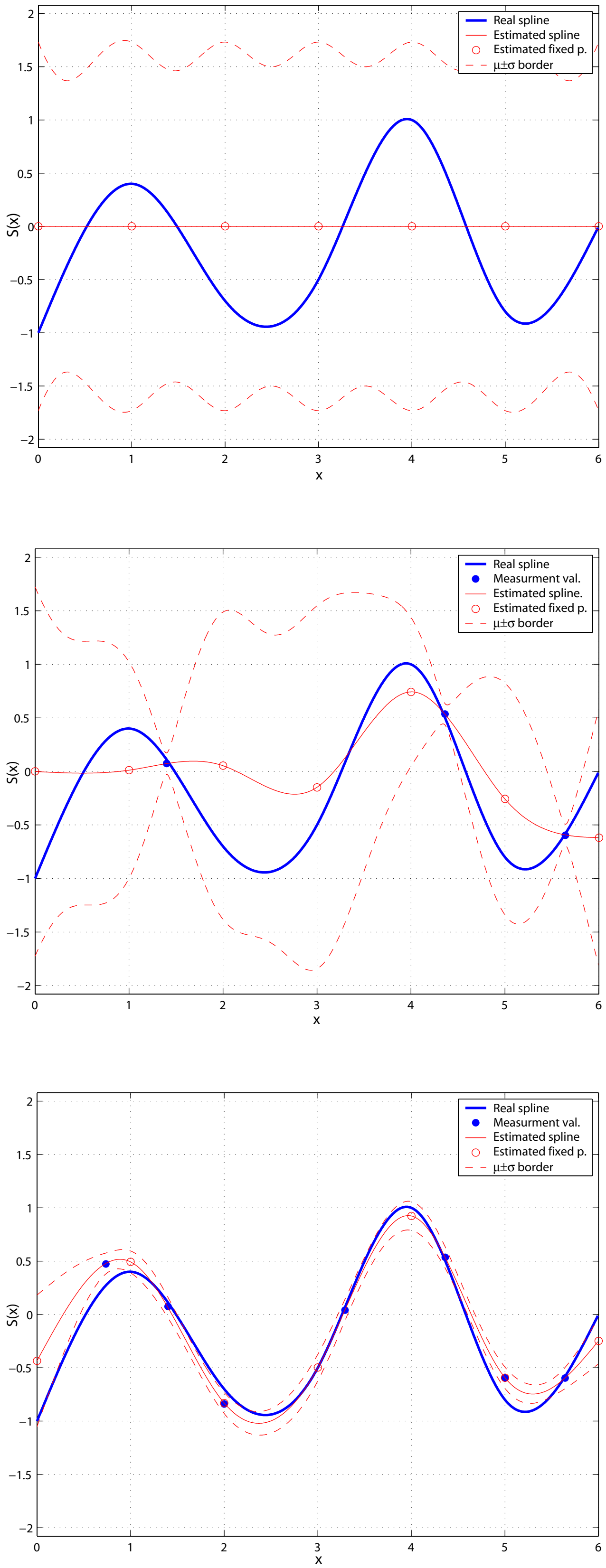

Fig. 4. Single Spline with 7 control points. The first graph shows the starting values, the middle one is after three measurements and last one is after seven measurements. The thick blue line is the real spline, the red thin one is the estimated spline and the red dashed lines are the $\hat{x}_p \pm \sigma_p^2$ borders. The blue filled dots are measurements, the red ones are the estimated control points. The units on the axes are arbitrary.

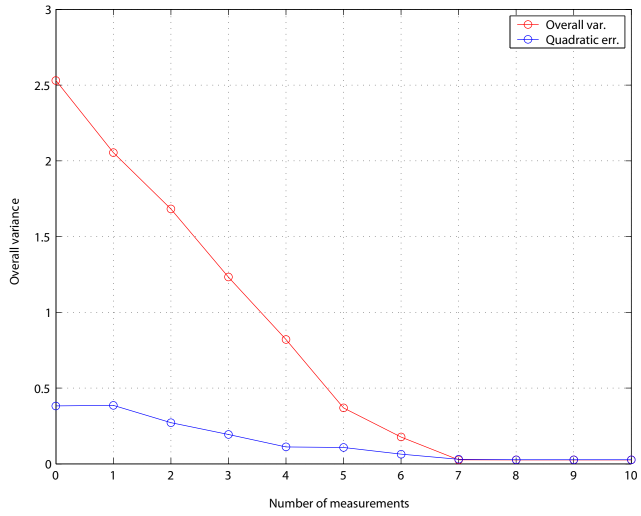

Fig. 5. Overall variance (upper red line) decrease until it reaches the expected value of squared error of every point (lower blue line).

V. Simulation Results

Figure 4 shows a measurement procedure of a single spline with seven control points. The real and estimated spline, as well as the estimated control points are shown. The filled blue points are the measurements, which have a variance $\sigma_v^2 = 0.01$. The variance of each point is visualized as $\hat{x}_p \pm \sigma_p^2$, which shows some oscillation. The reason being that the probability distribution functions in the control points were chosen to be independent Gaussians. The overall variance decreases almost linearly until the seventh measurement, as shown in figure 5. From the eighth measurement the decrease is much smaller. That result is expected, because with seven measurements, the measurements cover completely and optimally the state space (seven control points).

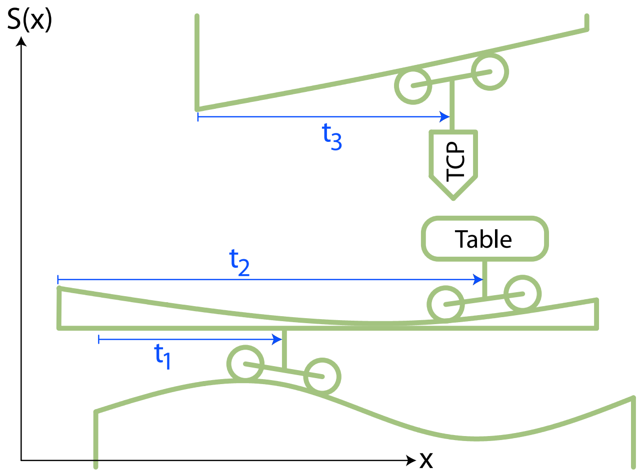

Fig. 6. One-dimensional sample tool machine. It has three linear joints t_1, t_2, t_3 in x-direction. Measured is the distance between table and TCP in $S(x)$ direction.

Our poorly constructed sample machine shown in

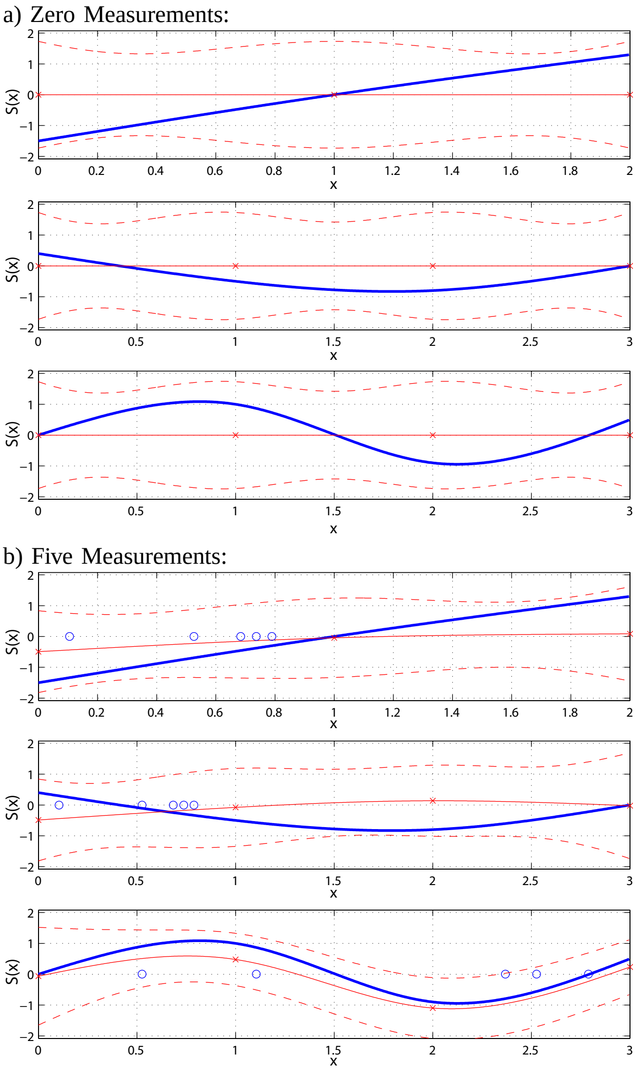

Fig. 7. Three stacked splines of the machine in figure 6 with a one dimensional measurement. The thick blue line is the real spline, red the estimated one. The red crosses mark the control points and the blue circles mark the measurement positions in x-direction. Red dashed lines are the $\hat{x}_p \pm \sigma_p^2$ borders.

figure 6 consists of three linear joints, stacked onto each other. Goal is to measure the mechanical deficiencies in the height, which are modeled as the sum of three splines as shown in figure 7. The height is the measurement with a smaller variance of $\sigma_v^2 = .0001$. It can easily be seen that after five measurements the estimate is much worse compared to the one-dimensional spline. On one hand this is due to larger number of control points and to the inconvenient geometry, since all three splines uncertainties always influence the measurement. On the other hand, at this point of time, it could not be ensured that numeric errors were negligible, when solving (27). On this point, investigations are continuing.

VI. Conclusion and Future Work

In this paper, we outlined the problems concerning calibration of modern machine tools. We showed that cubic splines can be used to efficiently model mechanical deficiencies of linear and rotary joints. We explained how to estimate the control points of splines out of measurements using stochastic combination. Deriving a weighting function for the quality of a spline estimate enabled us to determine optimal joint positions for subsequent measurements. These methods were combined to a framework for an optimal calibration procedure on machines with more than one joint. The framework assures that a predefined precision of a deficiency estimate can be achieved with a minimal number of measurements. So the time needed for calibrating machine tools can be drastically reduced. Another advantage is that for a large number of joints, the determination of the minimum of the weighting function, besides calculating the roots of a tenth order polynomial, can still be done analytically.

The main objective of future work is to allow non-linear joint errors and non-gaussian probability distributions as well as including costs which are not directly related to the precision of the estimate (e.g. time needed to reach a machine configuration). The use of a progressive estimation approach [10] to approximate the probability distributions will be investigated. A further point is to apply this framework to other types of splines. For example, for machine tools, bi-cubic splines could be used to model plane-errors.

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