Heart Phase Estimation Using Directional Statistics for Robotic Beating Heart Surgery

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***Abstract*—Robotic beating heart surgery requires accurate information about the current state of the heart. For this purpose, it is of great importance to have a good estimate of the heart's current phase, which in essence corresponds to the percentage of the current heart cycle that has already passed. Estimation of the heart phase is a highly nontrivial problem as the heart motion is not exactly periodic. On the contrary, it varies slightly from beat to beat and changes in frequency over time. In order to derive a robust phase estimation algorithm, we rely on directional statistics, a subfield of statistics that deals with quantities that are inherently periodic, such as the phase of the beating heart. The proposed methods are evaluated on a real data set and shown to be superior to the state of the art.**

***Keywords*—wrapped normal, periodicity, heart rate, blood pressure, expectation maximization**

I. Introduction

Every year, several million people die as a result of coronary artery disease. One of the treatments for this disease is the coronary artery bypass graft (CABG), a procedure where blood flow to the myocardium (the heart muscle) is restored by surgically creating a bypass of the affected blood vessels. As performing surgery on the beating heart is very demanding even for skilled surgeons, this procedure is commonly performed on a stopped heart. However, there are significant advantages for the patient if stopping the heart can be avoided, and thus, there has been a lot of work on robotics-based solutions for the problem of beating heart surgery in the past decade [1].

To address this problem, Nakamura et al. suggested the following concept for robotic beating heart surgery in 2001 [2]. The operation is carried out by a remote-controlled robot that automatically cancels out the motion of the beating heart. The surgeon who remotely controls the robot is, in turn, shown a stabilized image of the heart. This creates the illusion of operating on a still heart, even though the heart is, as a matter of fact, beating.

Automatic motion cancellation is, of course, founded on a highly accurate, reliable, and fast tracking of the movements of the beating heart. One of the most relevant quantities to describe the current state of the heart is its *phase*. In this context, the term phase can be explained as follows. Imagine that the heart performs a nearly (but not exactly) periodic movement with every heart beat. Now, based on measurements obtained from the heart using certain sensors, we seek to answer the question how much of the heart cycle has already passed and how much is yet to come. How far in the current heart beat are we at the moment? What percentage of the current heart beat has already passed? To answer these questions, we propose to employ a phase estimation algorithm.

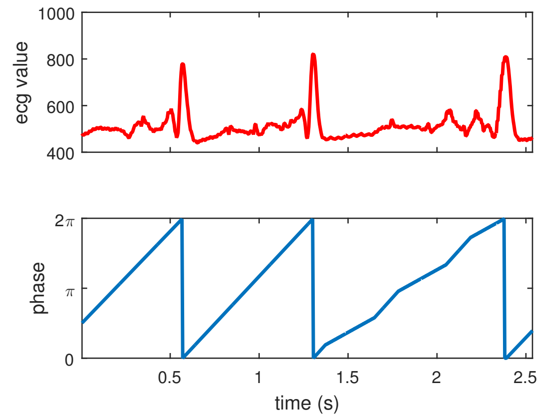


Fig. 1: Illustration of the phase of an ECG signal.

There is some work on phase estimation in literature, but it is usually applied to very different types of problems [3]. A very common application is the phase-locked loop (PLL), a control system that tries to track the phase of an input signal [4]. In the field of information theory and communication, the use of phase shift keying (PSK) modulation has been considered, which requires estimation of phase shifts in the received signal [5].

Phase estimation for the beating heart is a related but somewhat different problem. For many traditional phase estimation problems, the signal is assumed to be (almost) exactly periodic and it is sufficient to estimate the phase at the beginning of the signal to know the phase everywhere. By observing the signal for a certain amount of time, sufficient data for accurate phase estimation can be gathered. In the case of the beating heart, however, the movement is not exactly periodic and we need to estimate the phase at every point in time. Depending on the situation, the heart frequency and amplitude might change fairly quickly, especially during heart surgery. As even a single heart beat may be different from other heartbeats, for example as a result of arrhythmia (e.g., a premature ventricular contraction), an algorithm for heart phase estimation needs to be able to quickly adjust to changes in the heart movement (see Fig. 1).

The problem of phase estimation for the beating heart has only been considered by few authors. For example, Nascimento et al. have proposed a method for estimation based on echographic images [6]. However, their method is based on a multiple model filter and can only distinguish a finite number of phase values (two in their paper, systole and diastole). A method based on artificial neural networks that is also limited to distinguishing systole and diastole was proposed by Bibicu et al. [7]. Moreover, an approach based on computing local extrema in the image intensity was proposed by Hernàndez-Sabaté et al. [8].

In this paper, we consider a phase estimation approach based on directional statistics [9], [10], a subfield of statistics dealing with quantities on periodic manifolds. Unlike traditional approaches based on linear approximations, directional statistics is able to properly consider the inherent periodicity of the phase estimation problem. The application of directional statistics to the estimation of phase or phase difference has previously been considered by Stienne et al. for the case of GPS signals [11] and by Traa et al. in the context of microphone arrays [12]. However, the approaches by Stienne and Traa do not allow sensor fusion based on arbitrary likelihoods.

The approach proposed in this paper is based on the nonlinear circular filter that we proposed in [13] and extended in [14], [15]. This filter is based on the so-called wrapped normal distribution and can deal with nonlinear system and measurement equations. A more detailed discussion of this filter as well as some preliminary work on applying it to heart phase estimation can be found in [16].

The contributions of this paper can be summarized as follows. We propose a novel phase estimation algorithm, which is based on the application of the circular filter published in [17] to the problem of phase estimation. Furthermore, we present an expectation maximization algorithm for the partially wrapped normal distribution, which is, unlike previous approaches, based on moment matching rather than maximum likelihood estimation. Finally, we show how to apply the discussed algorithms to the problem of heart phase estimation and provide a thorough evaluation based on simulated as well as real data.

This paper is structured as follows. First, we introduce the concepts of periodicity and phase more rigorously in Sec. II. Then, we present the proposed phase estimation algorithm based on directional statistics in Sec. III and explain how to apply it to the problem of the beating heart in Sec. IV. Then, the proposed method is thoroughly evaluated in Sec. V. This paper is concluded in Sec. VI.

II. Periodicity and Phase

In this section, we introduce the concepts of periodicity and phase. Particularly, we consider the periodicity of functions that are not exactly periodic, but only approximately periodic in some sense. Then, we explain how the concept of phase can be applied in cases where the property of periodicity does not hold exactly. There is some literature on the topic of functions with non-exact periodicity (such as [18]) and the terms *quasiperiodic* and *approximately periodic* have been coined. In the following, we will introduce our own nomenclature, however, as the terms found in literature do not precisely specify the type of functions considered in this paper.

Let us first consider *exactly periodic functions*. A function $f : \mathbb{R} \to \mathbb{R}$ is called exactly periodic with period $\Delta t > 0$ if and only if

$$f(t) = f(t + \Delta t), \quad \forall t \in \mathbb{R} \ .$$

Examples of exactly periodic functions are $f(t) = \cos(t)$ with period $\Delta t = 2\pi$ and $f(t) = \mod(t, 2)$ with period $\Delta t = 2$. The concept of exact periodicity is illustrated in Fig. 2a.

One way to relax the definition of exactly periodic functions is to consider functions that are *approximately periodic in terms of value*. In this case, we do not require the function values in every period to be equal, but only similar. A function $f : \mathbb{R} \to \mathbb{R}$ is called approximately periodic in terms of value with period $\Delta t > 0$ if and only if

$$f(t) \approx f(t + \Delta t), \quad \forall t \in \mathbb{R} \ .$$

Functions of this type appear, for example, when an exactly periodic signal is superimposed by zero-mean additive noise. This concept is shown in Fig. 2b.

It is, however, also possible to relax the definition of exact periodicity in a different way by considering functions that are *approximately periodic in terms of time*. Here, we drop the requirement that the function values have to repeat with a constant period Δt. A function $f : \mathbb{R} \to \mathbb{R}$ is called approximately periodic in terms of time if and only if

$$f(t) = f(g(t + \Delta t)), \quad \forall t \in \mathbb{R} \ ,$$

where $g : \mathbb{R} \to \mathbb{R}$ is a continous and strictly increasing function. If we consider t as the time[1], this intuitively means that the time does not pass evenly, but can speed up or slow down depending on the function g. An illustration of this concept is given in Fig. 2c.

Furthermore, it is possible to consider a combination of both relaxations of exact periodicity. In this case, we have a function that is only *approximately periodic with respect to both value and time*. Functions of this type occur in a variety of real-world problems, particularly in the problem of heart phase estimation, which is considered in this paper. A function $f : \mathbb{R} \to \mathbb{R}$ is called approximately periodic in terms of both value and time if and only if

$$f(t) \approx f(g(t + \Delta t)), \quad \forall t \in \mathbb{R} \ ,$$

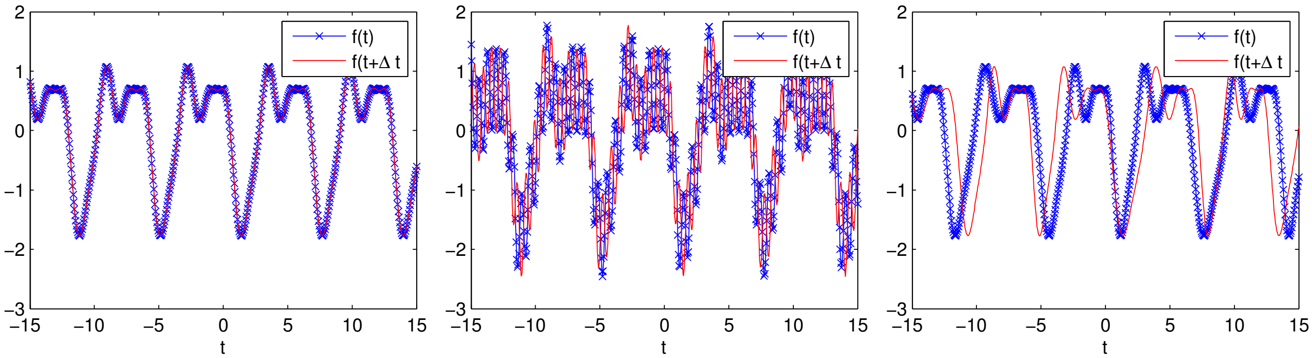
where $g : \mathbb{R} \to \mathbb{R}$ is a continous, bijective, and strictly increasing function.

Now we consider the definition of phase for these types of functions. Particularly, we consider the phase as a time-variant quantity $\phi(t) \in [0, 2\pi)$, which is sometimes also referred to as instantaneous phase or local phase. As the choice which point in time has phase $\phi(t) = 0$ is arbitrary, we define $\phi(0) = 0$. Then, the phase of an exactly periodic function $f(\cdot)$ with period Δt at time t is given by

$$\phi(t) = \frac{2\pi}{\Delta t} \cdot (t \bmod \Delta t) \in [0, 2\pi) \ .$$

Because this equation does not involve the value of $f(\cdot)$, it also applies to functions that are approximately periodic in

[1] It should be noted that these concepts can also be applied to cases where t is not time, for example if spatial periodicity is to be considered.



(a) Exactly Periodic. (b) Approximately periodic in terms of value. (c) Approximately periodic in terms of time.

Fig. 2: Examples for the different concepts of periodicity.

terms of value. For a function $f(\cdot)$ with period Δt that is only approximately periodic in terms of time, we have to transform the time using the inverse of $g(\cdot)$, before obtaining the phase. This yields

$$\phi(t) = \frac{2\pi}{\Delta t} \cdot (g^{-1}(t) \bmod \Delta t) \in [0, 2\pi) ,$$

which is obviously also applicable to functions that are only approximately periodic in terms of both value and time.

III. Phase Estimation Algorithm

Let us now consider a system whose state x_k at time step k is the phase at this point in time. In the following we will discuss an algorithm suitable for estimating the state x_k based on noisy measurements. As phase is a periodic quantity, we propose the use of an estimation algorithm based on directional statistics.

A. Directional Statistics

Before we discuss the filtering algorithm, we give a brief introduction into the underlying concepts of directional statistics.

Consider a real-valued random variable $x \sim \mathcal{N}(x; \mu, \sigma)$ distributed according to a normal distribution with mean μ and standard deviation σ. If we take $x \bmod 2\pi$, we obtain a circular random variable on $[0, 2\pi)$ and its distribution is given according to the following definition.

Definition 1 (Wrapped Normal Distribution)**.** A wrapped normal (WN) distribution with parameters $\mu \in [0, 2\pi)$ and $\sigma > 0$ is given by the probability density function

$$\mathcal{WN}(x; \mu, \sigma) = \sum_{j=-\infty}^{\infty} \mathcal{N}(x + 2\pi j; \mu, \sigma)$$

for $x \in [0, 2\pi)$.

The WN distribution arises as a limit distribution on the circle [13] and can be seen as a natural counterpart of the normal distribution for circular random variables.

An important concept in directional statistics are circular (or trigonometric) moments. For a circular random variable, the n-th trigonometric moment is given by $\mathbb{E}\left(e^{inx}\right) \in \mathbb{C}$, where i is the imaginary unit. For a WN-distributed random variable $x \sim \mathcal{WN}(x; \mu, \sigma)$, the n-th trigonometric moment can be calculated according to $m_n = \exp(in\mu - n^2\sigma^2/2)$.

In order to derive a filter with a nonlinear measurement update, it is helpful to approximate a continous WN density with a set of (weighted) samples. While it is possible to use stochastic samples for this purpose (similar to the Gaussian Particle Filter [19]), it has been shown that an intelligent deterministic algorithm for choosing samples representative of the original distribution produces more reliable results with a much smaller number of samples [15].

A deterministic sampling scheme for circular densities, particularly the WN density, has been presented in [20]. It is based on matching the first two trigonometric moments m_1 and m_2 to obtain a set of $L = 5$ weighted samples at positions $\beta_1, \ldots, \beta_L \in [0, 2\pi)$ with weights $\gamma_1, \ldots, \gamma_L > 0$ and $\sum_{l=1}^{L} \gamma_l = 1$. In the following, we will use this sampling technique. Conversely, we can easily estimate the parameters of a WN distribution from weighted samples by matching the first trigonometric moment. For L weighted samples at positions $\beta_1, \ldots, \beta_L \in [0, 2\pi)$ with weights $\gamma_1, \ldots, \gamma_L > 0$ and $\sum_{l=1}^{L} \gamma_l = 1$, we obtain $\mathcal{WN}(x; \mu, \sigma)$ with

$$\mu = \operatorname{atan2}\left(\sum_{l=1}^{L} \gamma_l \sin(\beta_l), \sum_{l=1}^{L} \gamma_l \cos(\beta_l)\right) ,$$

$$\sigma = \sqrt{-2\log\left(\sum_{l=1}^{L} \gamma_l \cos(\beta_l - \mu)\right)} .$$

B. Circular Filtering

Based on these fundamentals, it is possible to develop a recursive filtering algorithm. This algorithm consists of a prediction and an update step. The occurring distributions are assumed to be WN.

1) Prediction: In this paper, we assume that the system model is given by the equation

$$x_{k+1} = x_k + c_k + w_k \bmod 2\pi ,$$

where $w_k \sim \mathcal{WN}(w; \mu^w = 0, \sigma^w)$ is WN-distributed noise and $c_k \in [0, 2\pi)$ is a known offset. A nonlinear generalization was presented in [13], see also [15].

According to the Chapman–Kolmogorov equation, we obtain the predicted density $f^p(x_{k+1})$ according to

$$\begin{aligned}
&f^p(x_{k+1}) \\
&= \int_0^{2\pi} f(x_{k+1}|x_k) f^e(x_k) \, \mathrm{d}x_k \\
&= \int_0^{2\pi} \int_0^{2\pi} f(x_{k+1}|w_k, x_k) f^e(x_k) f^w(w_k) \, \mathrm{d}w_k \, \mathrm{d}x_k \\
&= \int_0^{2\pi} \int_0^{2\pi} \delta(x_{k+1} - (x_k + c_k + w_k) \bmod 2\pi) \\
&\quad f^e(x_k) f^w(w_k) \, \mathrm{d}w_k \, \mathrm{d}x_k \\
&= \int_0^{2\pi} f^e(x_{k+1} - c_k - w_k \bmod 2\pi) f^w(w_k) \, \mathrm{d}w_k \\
&= (f^e_{c_k} * f^w)(x_{k+1}) \ ,
\end{aligned}$$

where $f^e_{c_k}(x) = f^e(x - c_k \bmod 2\pi)$ is the shifted prior and $*$ indicates the convolution on the circle.

As we have shown in [15], the convolution of two WN densities with parameters $\mathcal{WN}(x; \mu_1, \sigma_1)$ and $\mathcal{WN}(x; \mu_2, \sigma_2)$ is given by $\mathcal{WN}(x; \mu_1 + \mu_2, \sqrt{\sigma_1^2 + \sigma_2^2})$.

2) Update: For the measurement update, we assume that a probabilistic measurement model in the form of the likelihood $f(\hat{z}_k|x_k)$ is given. According to Bayes' theorem, we have

$$f^e_{x_k} = f(x_k|\hat{z}_k) \propto f(\hat{z}_k|x_k) f^p(x_k) \ ,$$

i.e., the estimated density is given by the renormalized product of the likelihood and the prior. As this product is difficult to evaluate in closed-form, we use the sample-based approximation discussed above. For a known likelihood, it is easy to multiply the weight of each sample with the likelihood at that location and to estimate the parameters of the posterior WN from the reweighted samples by moment matching.

In practice, this approach suffers from the problem of sample degeneration, i.e., the weights of some (or all) samples are very close to zero after reweighting. To address this problem, we use a progressive technique initially proposed for the Gaussian case in [21]. A version of this method that was adapted to the circle is discussed in detail in [14], [15].

IV. Application to Heart Phase Estimation

In order to apply the phase estimation algorithm introduced above to a practical problem such as heart phase estimation, some additional steps are necessary. Particularly, we need to define the system model as well as the likelihood function in a suitable fashion.

First of all, we assume a discrete-time system model whose time step duration is equal to the sampling frequency ξ^s of the involved sensors[2]. Furthermore, as frequency estimation is out of the scope of this paper, we assume that the heart frequency ξ^h_k at time step k is (approximately) known.

[2]It is easy to generalize the methods discussed here to multiple sensor with different sampling frequencies.

Then, the system model is given by

$$x_{k+1} = x_k + 2\pi \cdot \frac{\xi^h_k}{\xi^s} + w_k \bmod 2\pi \ ,$$

with noise $w_k \sim \mathcal{WN}(w; \mu^w, \sigma^w)$.

In order to obtain the likelihood $f(z_k|x_k)$ defining the measurement model, we propose a data-driven approach. For this purpose, we consider $f(z_k|x_k)$ as a function of two variables[3], z_k and x_k. As discussed before, the state x_k is a directional quantity defined on the circle $[0, 2\pi)$. The measurement z_k, however, is typically not a directional quantity. In the following, we assume that the measurement is a real number, which is a realistic assumption for the measurements provided by most sensors such as electrocardiogram sensors, pressure sensors, and photoplethysmogram sensors. As a result, the likelihood can be described by a partially wrapped (unnormalized) density. For this reason, we consider the *partially wrapped normal distribution*.

Definition 2 (Partially Wrapped Normal Distribution)**.** The partially wrapped normal (PWN) distribution for one circular and one linear dimension is given by the probability density function

$$\mathcal{PWN}(\underline{x}, \underline{\mu}, \mathbf{C}) = \sum_{j=-\infty}^{\infty} \mathcal{N}\left(\underline{x} + [2\pi j, 0]^T; \underline{\mu}, \mathbf{C}\right) \ ,$$

where $\underline{x}, \underline{\mu} \in [0, 2\pi) \times \mathbb{R}$ and $\mathbf{C} \in \mathbb{R}^{2\times 2}$ is a symmetric positive definite matrix.

Some discussion on the case with one circular and two linear dimensions can be found in [22], [23], and the general case with an arbitrary number of circular and linear dimensions is treated in [16]. Similar to the way Gaussian distributions can be generalized to Gaussian mixtures, we now consider mixtures of PWN distributions. Roy et al. have also published some work on PWN mixtures, which they refer to as *semi-wrapped Gaussian mixture model (SWGMM)* [23]. A PWN mixture with L components is given by

$$\sum_{l=1}^{L} \omega_l \cdot \mathcal{PWN}(\underline{x}, \underline{\mu}_l, \mathbf{C}_l) \ ,$$

where $\omega_1, \ldots, \omega_L > 0$ with $\sum_{l=1}^{L} \omega_l = 1$ are weighting coefficients, and $\mathcal{PWN}(\underline{x}, \underline{\mu}_1, \mathbf{C}_1), \ldots, \mathcal{PWN}(\underline{x}, \underline{\mu}_L, \mathbf{C}_L)$ are PWN distributions.

When applying the proposed phase estimation algorithm to a real-world problem, one is faced with the question of how to obtain the likelihood function. In some cases a mathematical model of the system may be used to derive the likelihood. In the case of the beating heart, it might be possible to employ a complex physiological model of the heart such as [24] to derive the probability of a measurement $\hat{z}_k$ given a certain phase x_k. To avoid this complexity, we use a different approach in this paper, where we estimate the parameters of the PWN distribution based on samples obtained from labeled training data.

Estimation of mixture parameters is commonly done using the expectation maximization (EM) algorithm. We propose a

[3]The likelihood is commonly thought of as a function of x_k for fixed $\hat{z}_k$.

version of this algorithm adapted to the estimation of PWN mixture parameters. The proposed EM-like algorithm is somewhat unusual because it does not use maximum likelihood estimation (MLE) to obtain the parameters of the PWN distributions, but rather attempts to match hybrid moments, a generalized concept of moments for partially wrapped distributions introduced in [22]. The reason for this choice lies in the fact that MLE is not possible in closed-form (even for scalar WN distributions). As a result, approaches in literature such as [23], [25] use an approximate version of MLE instead, which leads to suboptimal results, while still being quite costly to compute.

Pseudo code of the resulting algorithm is given in Algorithm 1, which is based on the moment-based parameter estimation scheme given in Algorithm 2 (see also [16]).

Algorithm 1: Step of the EM-like algorithm for PWN mixtures.

Input: samples $\underline{x}_1, \dots, \underline{x}_N \in S^1 \times \mathbb{R}$, PWN mixture parameters $(\underline{\mu}_1, \dots, \underline{\mu}_L, \mathbf{C}_1, \dots, \mathbf{C}_L, \omega_1, \dots, \omega_L)$
Output: new PWN mixture parameters $(\underline{\mu}_1, \dots, \underline{\mu}_L, \mathbf{C}_1, \dots, \mathbf{C}_L, \omega_1, \dots, \omega_L)$

```
// E-Step
for n ← 1 to N do
    // assign sample n to component l
       with weight γ_{n,l}
    for l ← 1 to L do
        γ_{n,l} ← ω_l · PWN(x_n; μ_l, C_l, 1);
    end
    // normalize weights for sample n
    Γ_n ← Σ_{l=1}^{L} γ_{n,l};
    for l ← 1 to L do
        γ_{n,l} ← γ_{n,l}/Γ_n;
    end
end
// M-Step
for l ← 1 to L do
    // estimate parameters of component
       l from samples x_1,...x_N with
       weights γ_{1,l},...γ_{N,l}
    Γ_l = Σ_{n=1}^{N} γ_{n,l} ;
    (μ_l, C_l) ← paramEstim(x_1,...x_N, γ_{1,l}/Γ_l, ..., γ_{N,l}/Γ_l);
end
for l ← 1 to L do
    ω_l ← Γ_l / Σ_{l=1}^{L} Γ_l ;
end
return (μ_1, ..., μ_L, C_1, ..., C_L, ω_1, ..., ω_L);
```

The proposed method can be generalized to use multiple sensors by considering a likelihood $f(\underline{z}|x)$ with a vectorial measurement $\underline{z}$. Consequently, a PWN distribution of higher dimension has to be used (see [16]). In many cases, different sensors can be assumed to be independent, which allows factoring the likelihood as $f(\underline{z}|x) = f(z_1|x) \cdot \ldots \cdot f(z_s)$. Then, the estimate can be obtained by performing s successive measurement updates with scalar measurements, i.e., one measurement update per sensor.

Algorithm 2: Parameter estimation for PWN.

Input: samples $\underline{x}_1, \dots \underline{x}_N \in S^1 \times \mathbb{R}$, normalized weights $\gamma_1, \dots, \gamma_N > 0$
Output: PWN parameters $\underline{\mu}$, $\mathbf{C}$

```
// augment angular dimension
for n ← 1 to N do
    x̃_n ← [cos(x_{n,1}), sin(x_{n,1}), x_{n,2}]^T;
end
// calculate hyrid moments
μ̃ = Σ_{n=1}^{N} γ_n x̃_n;
C̃ = Σ_{n=1}^{N} γ_n (x̃_n − μ̃)(x̃_n − μ̃)^T;
// obtain PWN parameters
μ ← [atan2(μ̃_2, μ̃_1), μ̃_3]^T;
c_11 ← −2 log(sqrt(μ̃_1^2 + μ̃_2^2)) ;
c_12 ← exp(c_11/2)(−c̃_13 sin(μ_1) + c̃_23 cos(μ_1)) ;
c_22 ← c̃_33;
C ← [c_11 c_12; c_12 c_22] ;
return μ, C;
```

V. Evaluation

In this section, we evaluate the proposed methods on both simulated and real data. For the simulated data, we generated two signals, one for training and one for evaluation. The signal for training is given by

$$h(t) = \sin(\phi(t)) + v_t$$

with additive Gaussian noise $v_t \sim \mathcal{N}(v; 0, 0.2)$ and phase

$$\phi(t) = 2\pi \frac{\xi^h}{\xi^s} t \ ,$$

i.e., we have a fixed frequency of $\xi^h = 1.5\,\text{Hz}$. The signal for the evaluation is a sine function with the same amount of additive Gaussian noise, but it has the phase

$$\phi(t) = 2\pi \int_0^t \frac{\xi^h(\tau)}{\xi^s} \, \mathrm{d}\tau \ ,$$

where $\xi^h(t) = (1.5 + \cos(0.25t))$Hz is the time-varying heart frequency (see Fig. 3). The sampling frequency is $\xi^s = 1000\,\text{Hz}$ in both cases.

The real data set consists in a blood pressure signal recorded intraoperatively during coronary artery bypass graft surgery on a pig. The surgery was performed at the UniversitätsKlinikum Heidelberg (Heidelberg University Hospital). Once again, the sampling frequency is $\xi^s = 1000\,\text{Hz}$. In this paper, we consider four separate recordings (named 70, 77, 78, and 79), three of which (77,78,79) will be used for training and one of which (70, see Fig. 3) will be used for evaluation.

When applying the proposed methods to the real data, we use a preprocessing step to avoid certain issues. In reality, both the mean value and the amplitude (i.e., the difference between the minimum and maximum pressure in a single heart beat) change over time. In order to capture this effect, the measurement likelihood would need to be time-varying and depend on the current mean as well as amplitude. This can,

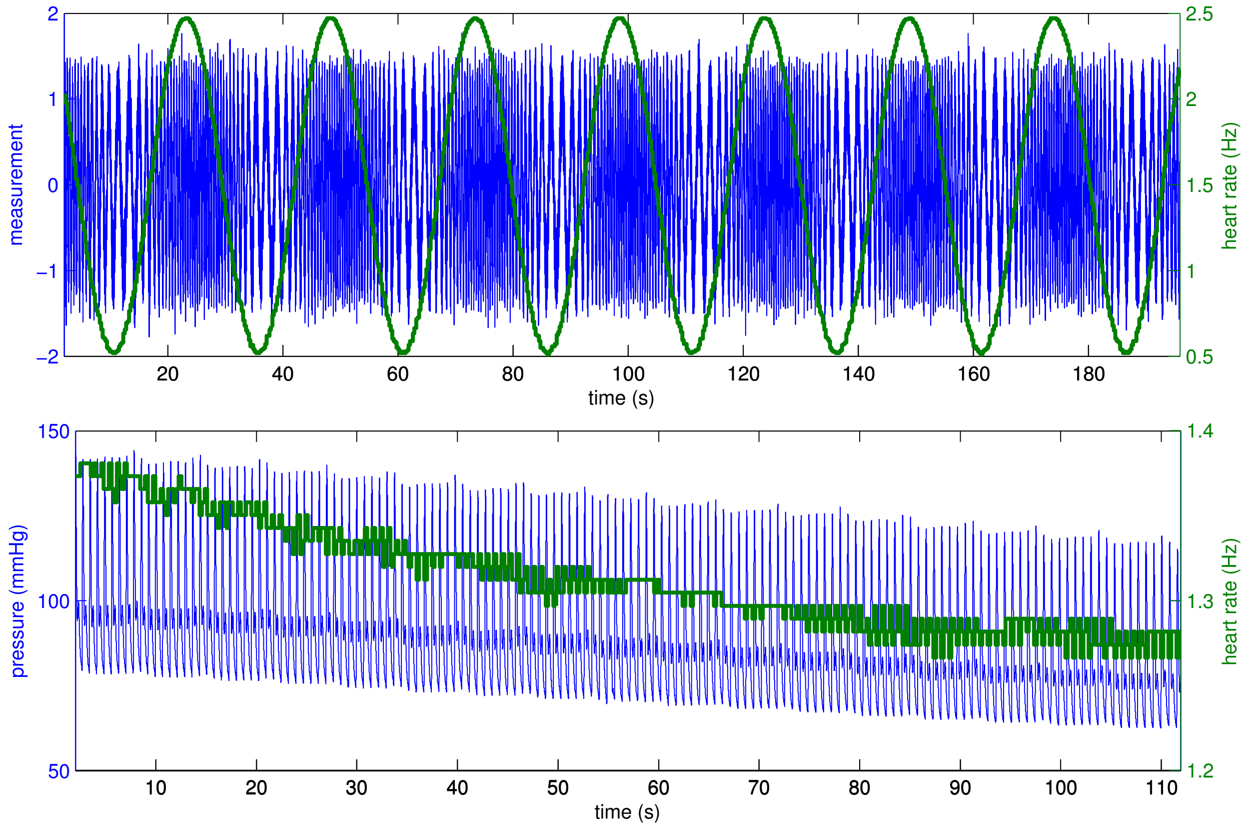
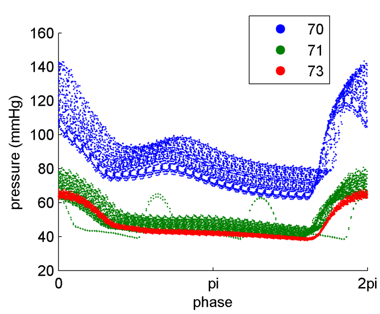
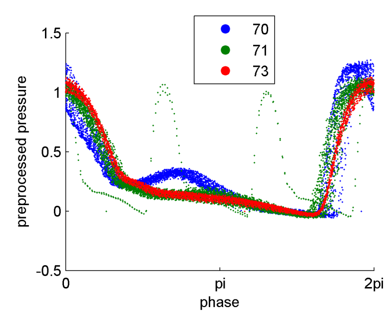


Fig. 3: Original pressure signal and heart rate obtained using STFT (top: simulated data, bottom: real data).



(a) Original data.



(b) Preprocessed data.

Fig. 4: Phase and pressure for real data.

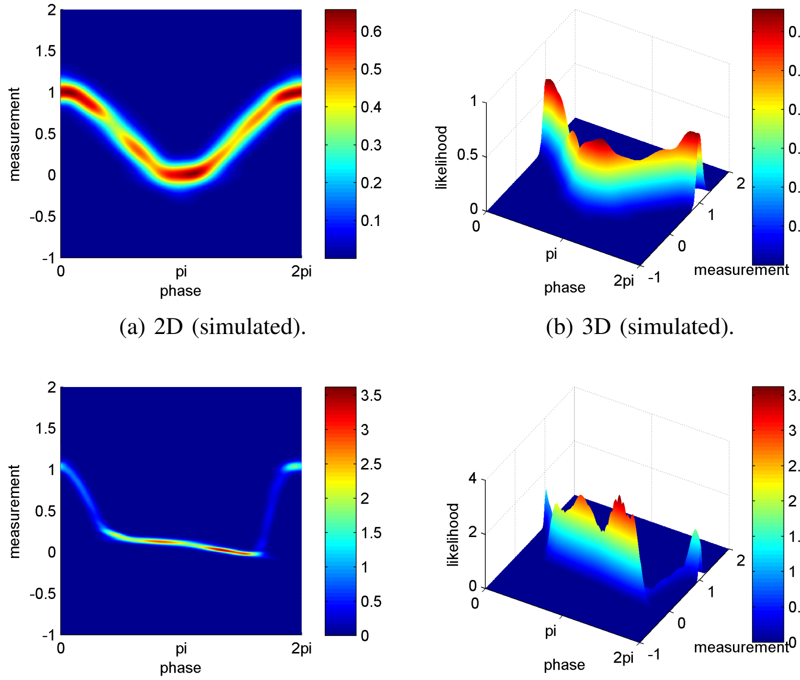
however, be avoided by considering the preprocessed signal given by

$$\hat{z}_k^{\text{preprocessed}} = \frac{\hat{z}_k - Q_k^{0,1}}{Q_k^{0.9} - Q_k^{0.1}}$$

where Q^p is the p-quantile of $z_{k-2000}, \ldots, z_k$. The effect of this step is depicted in Fig. 4. It can be seen as a normalization procedure that shifts and scales the raw measurements such that $80\,\%$ of the measurements are between 0 and 1.

We applied the EM-like algorithm introduced before to the simulated as well as the real data to obtain a PWN mixture model for the likelihood function. The model was initialized randomly and the number of components was chosen to be $L = 25$. The resulting likelihood functions are depicted in Fig. 5. The system noise was set to $\mathcal{WN}(x; 0, 0.03)$ for the simulated data and to $\mathcal{WN}(x; 0, 0.001)$ for the real data. A prediction is performed at every time step, i.e., every $1\,\mathrm{ms}$, and a measurement update is performed every ten time steps, i.e., every $10\,\mathrm{ms}$.

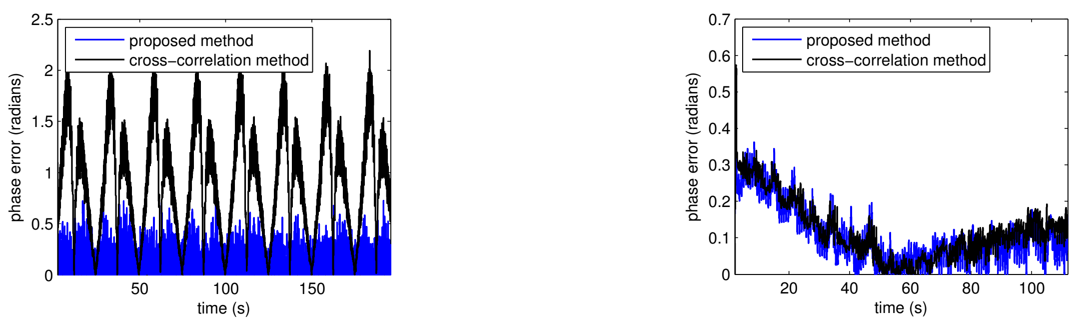
For comparison, we implemented an approach based on calculating the cross-correlation of the past measurements with a cosine function with the same frequency as the beating heart. The highest cross-correlation was obtained form a circular convolution within a window of $(\xi^h)^{-1}$ seconds, i.e., the length of one heartbeat.



(a) 2D (simulated). (b) 3D (simulated).

(c) 2D (real). (d) 3D (real).

Fig. 5: Likelihood functions obtained from simulated and real data visualized in 2D and 3D.



(a) Simulated data. (b) Real data.

Fig. 6: Phase error over time.

Because both approaches require a (rough) estimate of the current heart frequency, we implemented a short-time Fourier transform (STFT) with a window size of $4096\,\mathrm{ms}$. A new STFT was performed every $256\,\mathrm{ms}$ to obtain a current estimate of the heart frequency.

As an error measure, we use an angular version of the root-mean-square error (RMSE) given by

$$\sqrt{\frac{1}{k_{\max}}\sum_{k=1}^{k_{\max}} \min\left(|x_k - x_k^{\text{true}}|, 2\pi - |x_k - x_k^{\text{true}}|\right)^2}$$

to compare the estimated state with the ground truth for each time step. Intuitively, this error measure can be understood as an RMSE based on the geodetic distance on the circle, i.e, the shorter of the two connections between two points on the circle.

A constant offset in terms of phase does not really matter as the definition which phase corresponds to zero is arbitrary, we center the estimated phase values around the ground truth before calculating the error measure, i.e., we enforce that the estimate is unbiased.

The error of both approaches for the simulated and the real data set is shown in Fig. 6. For the simulated data, the total angular RMSE for the proposed approach is 0.1737, whereas the approach based on cross-correlation has angular RMSE of 1.0340. These results suggest that the proposed filter is far superior to the approach based on cross-correlation in challenging scenarios where the frequency changes rapidly.

On the real data set, the angular RMSE is given by 0.1311 and 0.1455, respectively, i.e., the error of the approach based on cross-correlation is approximately $11\,\%$ higher than the error of the proposed approach. The reason why the difference between the two methods is much smaller on the real data set is the fact that there are no rapid changes in frequency and no premature ventricular contractions in the considered signal. In other words, the real data set is not particularly challenging, and, as a result, even the approach based on cross-correlation has little trouble determining the correct phase. In future work, it would be interesting to consider more challenging signals.

VI. Conclusion

A novel phase estimation algorithm based on directional statistics has been presented in this paper. The proposed algorithm has been applied to the problem of heart phase estimation and can be employed in the context of robotic beating heart surgery.

In a challenging simulated setting, the evaluation shows far superior results compared to a standard approach. An application to real data indicated that the proposed algorithm works in real-life scenarios and performs better than the considered standard approach.

The proposed method can be extended in several ways. First, it would be interesting to consider the problem of jointly estimating the phase and the frequency of the signal. As phase is periodic but frequency is linear, this would necessitate a filter based on circular-linear distributions, e.g., the PWN distribution discussed in this paper. Phase and frequency are, obviously, not independent quantities, so it would be of particular importance to properly consider the involved circular-linear correlation. Second, the proposed method could be applied to a larger variety of sensors, possibly fusing measurements from different sources to obtain a better estimate. Third, the heart phase estimation algorithm could be included in a system for robotic beating heart surgery.

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