Bearings-Only Sensor Scheduling Using Circular Statistics

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***Abstract*—In this paper, we introduce a novel approach for scheduled tracking of a moving target based on bearings-only sensors. Unlike classical approaches that are typically based on the extended or unscented Kalman filter, we rely on circular statistics to describe probability distributions for angular measurements more accurately. As the energy available to sensors is limited in many scenarios, we introduce a scheduling algorithm that selects a subset of two sensors to be active at any given time step while minimizing the uncertainty of the state estimate. This is done by anticipating possible future measurements. We evaluate the proposed method in simulations and compare it to an UKF-based solution. Our evaluation demonstrates the superiority of the presented approach, particularly when high measurement uncertainty makes consideration of the circular geometry necessary.**

I. Introduction

Bearings-only sensors present a typical application for nonlinear estimation techniques. This sensor type appears in many real-world systems, such as certain passive radars, which might be either stationary or mobile. The need for tracking based solely on angular measurements arises when no distance information is given or when the given distance information is discarded due to high noise. A network of such angular sensors can be used in order to optimize estimation quality by performing several measurements simultaneously. Often such sensors use an autonomous and limited power supply, which limits the number of possible measurements. This scenario involves two important algorithmic problems. First, it is necessary to fuse prior knowledge and measurements into a good estimate of the true target position. Second, a measurement control sequence has to be found in order to optimally schedule the measurements optimizing the estimation quality, while reducing energy consumption.

Consideration of noise in angular measurements is usually based on a Gaussian assumption. This is particularly the case, when using well-established nonlinear estimation techniques such as the extended Kalman filter (EKF) or the unscented Kalman filter (UKF) [1]. The motivation of the widespread use of the normal distribution involves the central limit theorem. That is, the distribution of a renormalized sum of i.i.d. random variables with finite variance converges to a normally distributed random variable. This theorem makes the assumption

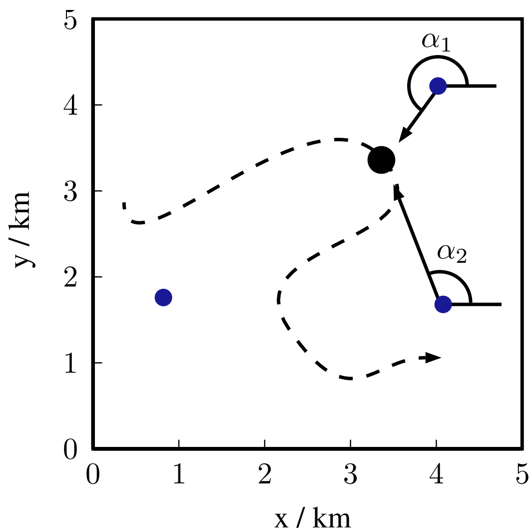


Fig. 1: In our considered scenario, only two of the sensors (depicted as blue dots) are measuring simultaneously. The target moves along a trajectory simulated using a constant velocity model.

of Gaussian noise in many applications plausible. However, it does not hold in a circular setting, because the normal distribution is defined on $\mathbb{R}$ and thus, it is not restricted to the circle. In estimation problems involving large uncertainty, this approximation might yield unfeasible results, because the periodic nature of the uncertainty is not taken into account by the normal distribution. Thus, in the case of bearings-only measurements a better performance can be achieved by considering probability distributions defined on the circle.

In this paper, we consider a scenario of multiple stationary bearings-only sensors in the Euclidean plane, which are used to track a moving object (see Fig. 1) and yield highly uncertain measurements. In order to reduce the number of measurements, the scheduler needs to choose those two sensors in each time step that minimize the expected uncertainty of the system state averaged over a given pre-defined time horizon. Circular statistics are used for modeling the measurement noise of the angular sensors. This is done by a deterministic approximation with a mixture of Dirac delta components, which outperforms approaches approximating the Gaussian distribution, particularly for very noisy sensors. The measurement uncertainty of a certain combination of sensors depends on the true system state. Thus, future measurements need to be predicted within the scheduling process. A schedule needs to be generated at each time step.

A. Related Work

Results related to this work originate from three areas of research. First, some recent results were made related to the methodology used in this paper. This involves both, recursive filtering of angular data based on directional statistics and deterministic approximation of probability distributions by a mixture of Dirac delta components. Second, there exists a broad discussion and research on signal processing and estimation based on bearings-only sensors. Finally, the problem of sensor scheduling and resource management in sensor networks is broadly investigated.

A comprehensive treatment of the field of circular and directional statistics is given in the books by Jammalamadaka and Sengupta [2] and by Mardia and Jupp [3]. They describe how statistical inference in a circular setting differs from classical approaches. Applying results from directional statistics for developing a recursive filter was done in [4], where a filter based on the von Mises distribution is derived. Later, in [5] a filter is proposed, which is based on matching the von Mises and the wrapped normal distribution and approximating them by a mixture of three Dirac delta components. Such approximations have already been investigated for the Gaussian distribution in [6], [7], and [8].

Several different approaches are considered in recent literature on bearings-only tracking. In [9], a sliced Gaussian mixture based filter is applied to cooperative passive target tracking. A maximum-likelihood approach to bearings-only tracking is given in [10]. In [11], multiple switching models are considered for target dynamics. Discussions of shifted Rayleigh and filters particle filters are given in [12], [13]. The latter suffer from the curse of dimensionality in high dimensional state spaces. A further analysis of bearings-only tracking can also be found in [14], [15].

The literature on sensor scheduling can be divided into two groups. First, linear estimation problems are considered, where system and measurement noise are Gaussian. In this situation, Kalman filters are applied for optimal estimation. Furthermore, the uncertainty of the estimate does not depend on the actual measurements, thus the schedule can be precalculated offline. Some of the works related to this approach are [16], [17], [18]. Second, scheduling scenarios involving nonlinear estimation problems are considered. Thus, it can not be generally assumed that the uncertainty of the estimate does not depend on the measurements. In this situation, approaches based on Partially Observable Markov Decision Processes (POMDPs) are commonly used. Literature involving scheduling and control of bearings-only measurement systems

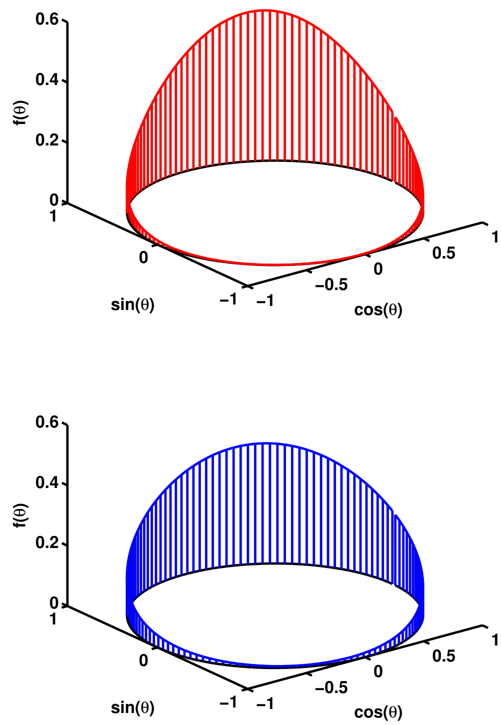


Fig. 2: Density of two wrapped normal distributions with the same mean and different dispersion parameters σ. The upper density has $\sigma = 0.8$ and the lower has $\sigma = 1$.

includes [19], [20], [21] and is usually based on a Gaussian distribution assumption at some point, which does not take the circular nature of angular data into account.

B. Key Idea and Outline

In our work, the moving object that needs to be tracked is described by a linear constant velocity model with Gaussian system noise. We propose modeling the measurement noise of the sensors with the wrapped normal distribution. This distribution is defined on the $SO(2)$ (the group of rotations in $\mathbb{R}^2$) and appears as a limit distribution for a circular central limit theorem. A method for approximating the wrapped normal distribution by a mixture of Dirac delta components is proposed. This is used to fuse the measurements of two sensors into a joint measurement of the true object position. This new joint measurement is computed by a combination of the earlier computed deterministic samples of the sensor noise from each participating sensor. The mean and covariance of this joint measurement are computed and used for a Kalman Filter measurement update.

The scheduling algorithm is based on predicting possible future measurements based on the system model. It computes the uncertainty of a possible measurement for each considered sensor combination and uses this for computing the uncertainty after a possible measurement update. For each possible measurement sequence the traces of the estimated covariance after each anticipated update step are added up. This comes down to a tree search, which is optimized by a classical branch and bound technique.

In the next section, we present some mathematical results from directional statistics. Particularly, the wrapped normal distribution and a deterministic Dirac mixture approximation based on circular moment matching are introduced. In Sec. III, the estimation and scheduling process is derived. The method used for processing and fusing the measurements and the branch and bound based algorithm for scheduling the sensors is presented. A comparison to a UKF based approach is given in Sec. IV. The work is concluded in Sec. V.

II. Circular Statistics and the Wrapped Normal Distribution

The wrapped normal distribution is the circular equivalent of the normal distribution. As the name suggests, it is obtained by wrapping the probability density of the normal distribution around the interval $[0, 2\pi)$. The importance of this distribution is due to the fact that it appears as a limit distribution for certain circular random variables. We will present some properties of the wrapped normal distribution and a method for deterministic Dirac mixture approximation of a wrapped normal random variable.

A. Wrapped Normal Distribution

Definition 1. *The probability distribution defined by the pdf*

$$f(\theta) = \frac{1}{\sigma\sqrt{2\pi}} \sum_{k=-\infty}^{\infty} \exp\left(\frac{-(\theta - \mu + 2\pi k)}{2\sigma^2}\right)$$

on $[0, 2\pi)$ with parameters $\mu \in [0, 2\pi)$ and $\sigma \in \mathbb{R}^+$ is called wrapped normal distribution. We will denote it by $\mathrm{WN}(\mu, \sigma)$.

The parameters of this distribution have a similar interpretation as in the classical Euclidean case, that is μ is a location parameter and σ is a dispersion parameter. However, it is important to note that in general, the expectation value is given by μ, but the covariance is not given by σ. Examples for wrapped normal probability densities are shown in Fig. 2. In most applications involving circular random variables the consideration of classical moments is not meaningful. Usually one considers circular moments on the unit disk in the complex plane. The n-th circular moment is defined by

$$\mathbb{E}\left(\mathrm{e}^{inX}\right) = \mathrm{e}^{in\mu - n^2\sigma^2/2} \in \mathbb{C} ,$$

where $X \sim \mathrm{WN}(\mu, \sigma)$.

A normal random variable X can be transformed into a wrapped normal random variable by computing $X \mod 2\pi$. This approach can be used for motivating a central limit type theorem for the circle [2]. Consider i.i.d. random variables θ_i defined on $[0, 2\pi)$ with $\mathbf{Var}(\theta_i) = 1$ and $\mathbb{E}(\theta_i) = \pi$, then

$$S_n := \frac{1}{\sqrt{n}} \sum_{i=1}^{n} (\theta_i - \pi) \xrightarrow{d} X , \quad n \to \infty ,$$

where $\xrightarrow{d}$ denotes convergence in distribution and $X \sim \mathcal{N}(0, 1)$. Thus

$$S_n \mod 2\pi \xrightarrow{d} \mathrm{WN}(0, 1) .$$

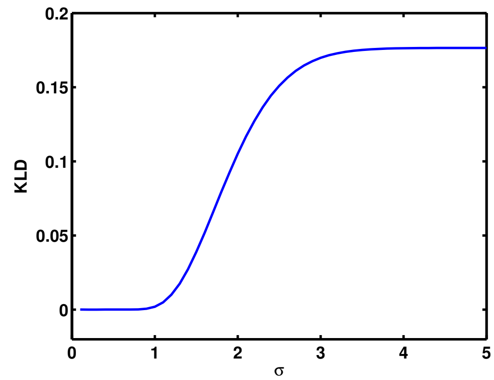


Fig. 3: Kullback-Leibler divergence when approximating a Wrapped-Normal distribution (defined on $[0, 2\pi)$) having mean π and dispersion parameter σ with a one dimensional normal distribution with same first and second moments.

Fig. 3 shows the approximation error made (in terms of Kullback-Leibler divergence) when a wrapped normal distribution is approximated by a normal distribution with same first and second moment, when the mean is placed optimally.

B. Dirac Mixture Approximation of Wrapped Normal Distribution

Dirac mixture approximation of wrapped normal distributions is based on moment matching as described in [5]. The $\mathrm{WN}(\mu, \sigma)$ distribution is approximated by a symmetric wrapped Dirac mixture consisting of $2k + 1$ components. To describe this distribution, $k + 1$ parameters are needed, which are denoted by $\mu, \alpha_1, \ldots \alpha_k \in [0, 2\pi)$. Thus, we obtain

$$\begin{aligned} f_d(\theta) =& \frac{\delta(\theta - \mu)}{2k + 1} + \sum_{i=0}^{k} \frac{\delta(\theta - (\mu + \alpha_i))}{2k + 1} \\ &+ \sum_{i=0}^{k} \frac{\delta(\theta - (\mu - \alpha_i))}{2k + 1} . \end{aligned}$$

Matching circular moments of both distributions yields

$$\begin{aligned} \exp\left(in\mu - \frac{n^2\sigma^2}{2}\right) =& \frac{\exp(in\mu)}{2k + 1} \\ &+ \sum_{i=0}^{k} \frac{\exp(in(\mu + \alpha_i))}{2k + 1} \\ &+ \sum_{i=0}^{k} \frac{\exp(in(\mu - \alpha_i))}{2k + 1} . \end{aligned}$$

In the case of $k = 1$, we match the first circular moment ($n = 1$) and solve for α_1

$$\frac{3}{2}\exp\left(-\frac{\sigma^2}{2}\right) - \frac{1}{2} = \cos(\alpha_1) \ .$$

This yields

$$\alpha_1 = \arccos\left(\frac{3}{2}\exp\left(-\frac{n^2\sigma^2}{2}\right) - \frac{1}{2}\right) \ . \qquad (1)$$

In contrast to the UKF, which samples the normal distribution, the presented approximation considers the circular nature of angular data. It can also be generalized to a higher number of samples for increasing estimation accuracy.

III. Scheduling Bearings-Only Sensors

The proposed sensor scheduling scheme reduces the number of measurements by using only two sensors at each time step. This is done by considering the estimated covariance of the tracked object after a measurement update, which requires a prediction of possible measurements and their respective covariances. The estimation itself is based on sampling the sensor noise deterministically and thus, predicting possible target positions in the Euclidean plane. The probabilistic model describing the sensor noise is based on the wrapped normal distribution.

A. System Model and the Prediction Step

The movements of the tracked object are described by a constant velocity model. That is, the system state is given by

$$\underline{x}_t = \begin{pmatrix} \underline{p} \\ \dot{\underline{p}} \end{pmatrix} \ ,$$

where $\underline{p} \in \mathbb{R}^2$ is the position of the tracked object and $\dot{\underline{p}}$ its velocity making $\underline{x}_t$ a four dimensional state vector. The system dynamics are described by

$$\underline{x}_{t+1} = \underbrace{\begin{pmatrix} \mathbf{I}_2 & \Delta t \cdot \mathbf{I}_2 \\ 0 & \mathbf{I}_2 \end{pmatrix}}_{=:\mathbf{A}} \underline{x}_t + \underline{w}_t \ .$$

where $\mathbf{I}_2$ is a 2×2 identity matrix and $\Delta t > 0$ denotes the duration of a time step. The system noise is assumed to be Gaussian, that is $\underline{w}_t \sim \mathcal{N}(\underline{0}, \mathbf{R})$ with noise covariance $\mathbf{R}$. As usually the filtered estimate of the true system state is denoted by $\underline{x}_t^e$ and its uncertainty is denoted by $\mathbf{C}_t^e$. Analogously $\underline{x}_t^p$ and $\mathbf{C}_t^p$ are used to denote the system state after the prediction step and the corresponding covariance. The prediction step is carried out using the classical Kalman filter formulas

$$\begin{aligned} \underline{x}_{t+1}^p &= \mathbf{A}\underline{x}_t^e \ , \\ \mathbf{C}_{t+1}^p &= \mathbf{A}\mathbf{C}_t^e\mathbf{A}^T + \mathbf{R} \ . \end{aligned}$$

Input: Planning horizon T; Bound for costs b;
Current state estimate $\underline{x}_t^e, \mathbf{C}_t^e$;
Output: Schedule $\underline{r}$; Cost c
$[x_{t+1}^p,\ \mathbf{C}_{t+1}^p] \leftarrow$ predict($\underline{x}_t^e, \mathbf{C}_t^e$);
`// Predict measurements of` x_{t+1}^p.
$\underline{\alpha} \leftarrow$ anticipateMeasurementAngles(x_{t+1}^p);
`// Lowest cost so far`
$c \leftarrow \infty$;
`// Try all sensor combinations`
foreach $i, j \in \{1, \dots, m\}, i \neq j$ **do**
    `// Compute Euclidean measurement`
    $[\underline{z}_{t+1}\mathbf{R}_{t+1}] \leftarrow$ joinMeasurements($i, j, \underline{\alpha}$);
    $[x_{t+1}^e,\ \mathbf{C}_{t+1}^e] \leftarrow$ update($\underline{x}_t^p, \mathbf{C}_t^p, \underline{z}_{t+1}, \mathbf{R}_{t+1}$);
    $v \leftarrow \mathrm{tr}(\mathbf{C}_{t+1}^e)$;
    if $v \leq b$ *and* $T = 1$ **then**
        `// Only one time step left`
        $c \leftarrow v$;
        $b \leftarrow v$;
        $r \leftarrow (i, j)^T$;
    else if $v \leq b$ *and* $T > 1$ **then**
        `// Calculate schedule recursively`
        $[\underline{s}, d] \leftarrow$ schedule($T - 1, b - v, \underline{x}_{t+1}^e, \mathbf{C}_{t+1}^e$);
        if $d < \infty$ **then**
            $c \leftarrow v + d$;
            $b \leftarrow c$;
            $\underline{r} \leftarrow [(i, j)^T\ \underline{s}]$;
        end
    end
end

Fig. 4: Algorithm for the scheduler.

B. Sensor Model and Measurement Update

We consider m sensors measuring true bearings, that is an angle relative to the north pole of a map. At each time step, two of this n sensors perform a measurement. Furthermore, the sensors are assumed to be noisy. Thus

$$\alpha_{i,t} = h_i(\underline{x}_t) + \omega_{i,t}$$

yields the angle between the sensor $i \in \{1, \dots, m\}$ and the target relatively to the north pole. The noise $\omega_{i,t}$ follows a wrapped normal distribution, that is $\omega_{i,t} \sim \mathrm{WN}(0, \sigma_i)$. The sensors are stationary and their positions are known exactly. They are given by the columns of the matrix

$$\mathbf{S} = (\underline{s}_1 \ \dots \ \underline{s}_m) \ .$$

In this setting, two operations are of interest. First, transforming measurements of two sensors into a position on the Euclidean plane, which will be used when measurements are processed. Second, predicting the measurement of a sensor, when the position is known, which will be necessary during the scheduling procedure.

The overall strategy for the measurement update step is as follows. In the first step, the noise density of the measuring sensors is approximated by a mixture of Dirac delta components in a deterministic way. This is done by (1). The approximated densities are placed around the measurements of the respective sensors and can be interpreted as uncertainty about the true measurement source. Each Dirac component of a measuring sensor is combined with each Dirac component of the other measuring sensor into a coordinate on the Euclidean plane. Thus, a sample of possible measurement sources is obtained. Finally, the mean and covariance of this sample is computed and it is used as input for a Kalman filter measurement update.

After the active sensors i, j have measured $\alpha_{i,t}$ and $\alpha_{j,t}$, these measurements are transformed into directional unit vectors according to

$$\underline{e}_{i,t} = \begin{pmatrix} \cos(\alpha_{i,t}) \\ \sin(\alpha_{i,t}) \end{pmatrix} \ , \quad \underline{e}_{j,t} = \begin{pmatrix} \cos(\alpha_{j,t}) \\ \sin(\alpha_{j,t}) \end{pmatrix} \ .$$

For finding the measurement source on the Euclidean plane, it is necessary to compute $a_t, b_t \in \mathbb{R}$ such that $\underline{s}_1 + a_t \underline{e}_1 = \underline{s}_2 + b_t \underline{e}_2$ Thus, we define the matrix $\mathbf{E} = (-\underline{e}_1, \ \underline{e}_2)$. Using this definition a_t and b_t can be computed by

$$\begin{pmatrix} a_t \\ b_t \end{pmatrix} = \mathbf{E}^{-1}(\underline{s}_1 - \underline{s}_2) \ .$$

Let $\underline{y}_t$ denote the joined measurement, that is

$$\underline{y}_t := \underline{s}_i + a_t \underline{e}_{i,t} = \underline{s}_j + b_t \underline{e}_{j,t} \ . \tag{2}$$

Now, we have

$$\begin{aligned} f(\underline{y}_t) &= \int_0^{2\pi} \int_0^{2\pi} f(\underline{y}, \alpha_{i,t}, \alpha_{j,t}) \ \mathrm{d}\alpha_{i,t} \ \mathrm{d}\alpha_{j,t} \\ &= \int_0^{2\pi} \int_0^{2\pi} f(\underline{y} \,|\, \alpha_{i,t}, \alpha_{j,t}) \cdot f(\alpha_{i,t}, \alpha_{j,t}) \ \mathrm{d}\alpha_{i,t} \ \mathrm{d}\alpha_{j,t} \ . \end{aligned}$$

In order to approximate $f(\underline{y}_t)$ by a normal distribution, we approximate $f(\alpha_{i,t}, \alpha_{j,t})$ by a mixture of Dirac delta components. For each Dirac pair $(\alpha_{i,t}, \alpha_{j,t})$, we compute a possible value for $\underline{y}_t$ using (2). That is, we obtain nine deterministic samples representing possible values of $\underline{y}_t$. The mean of these samples is taken as our joined measurement $\underline{y}_t$ and their empirical covariance is taken as our predicted uncertainty of the joined measurement $\mathbf{Q}_t^p$. Be aware that, while assuming a wrapped normal noise for our sensors, we assume a Gaussian uncertainty for the measured position of the tracked object. This approach considers the circular nature of sensor noise for the computation of $\underline{y}_t$ and $\mathbf{Q}_t^p$, while simultaneously preserving the elegance of a Gaussian uncertainty model. After this transformation, a linear measurement equation can be used

$$\underline{y}_t = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}}_{=:\mathbf{H}} \underline{x}_t^p + \underline{v}_t \ ,$$

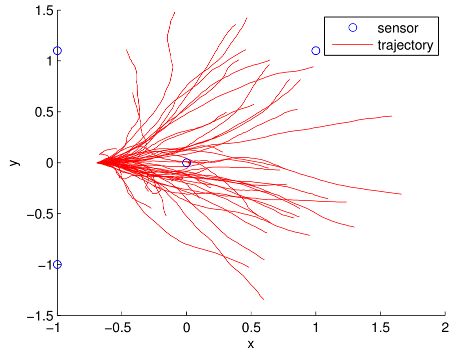


Fig. 5: Example trajectories.

where $\underline{v}_t \sim \mathcal{N}(\underline{0}, \mathbf{Q}_t^p)$. Now, the Kalman filter can be used for the measurement update step

$$\begin{aligned} \underline{z}_t &= \underline{y}_t - \mathbf{H}\underline{x}_t^p \ , & \mathbf{P}_t &= \mathbf{H}\mathbf{C}_t^p\mathbf{H}^T + \mathbf{Q}_t^p \ , \\ \mathbf{K}_t &= \mathbf{C}_t^p \mathbf{H}_t^T \mathbf{P}_t^{-1} \ , & \underline{x}_t^e &= \underline{x}_t^p + \mathbf{K}_t \underline{z}_t \ , \\ \mathbf{C}_t^e &= (\mathbf{I} - \mathbf{K}_t\mathbf{H})\mathbf{C}_t^p \ . \end{aligned}$$

C. Scheduling Bearings-Only Sensors

Scheduling our sensors is based on a branch-and-bound technique. The scheduling algorithm is described in Fig. 4, where the system model and sensor properties (i.e., the positions $\mathbf{S}$ and the noises σ_i) are assumed to be global variables and known to every subroutine. It predicts a future position and the measurements of this position including its uncertainties. Furthermore, a filter step is simulated. Anticipating a measurement of sensor j, when the true position is $\underline{x} = (x^{(1)}, x^{(2)})^T$, is done by computing

$$\mathrm{atan2}(s_j^{(2)} - x^{(2)}, \ s_j^{(2)} - x^{(2)}) \ .$$

The algorithm solves

$$\underline{r}_t = \arg\min_{\underline{r}_t} \left(\sum_{i=1}^{T} \mathbb{E}(\mathrm{tr}(\mathbf{C}_{t+i}^e) \,|\, \underline{r}_t) \right) \ ,$$

where $\underline{r}_t$ is a sensor schedule for a planning horizon T created at time step t and the expectation is taken over possible future system states.

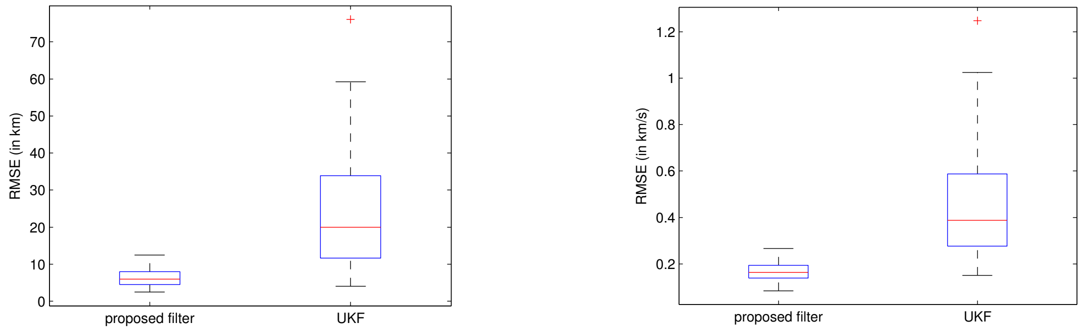
IV. Simulations

To evaluate the proposed algorithm we have performed several simulations. All distances are given in kilometers, all time intervals in seconds and all angles in radians.

For comparison, we implemented an unscented Kalman filter (UKF) [1] to perform the nonlinear measurement update. The UKF is based on the measurement equation

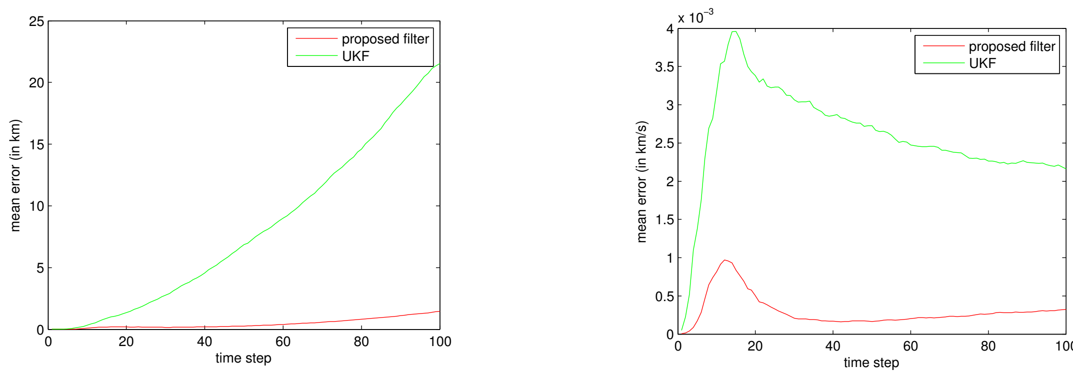
$$\begin{pmatrix} \alpha_{i,t} \\ \alpha_{j,t} \end{pmatrix} = \begin{pmatrix} \mathrm{atan2}(s_i^{(2)} - x_t^{(2)}, s_i^{(1)} - x_t^{(1)}) + v_{i,t}) & \mod 2\pi \\ \mathrm{atan2}(s_j^{(2)} - x_t^{(2)}, s_j^{(1)} - x_t^{(1)}) + v_{j,t}) & \mod 2\pi \end{pmatrix}$$

if sensor i and j are selected in time step t. In order to avoid issues when dealing with angles near the discontinuity between



(a) Position. (b) Velocity.

Fig. 7: Boxplots of the RMSE.



(a) Position. (b) Velocity.

Fig. 8: Mean of the RMSE.

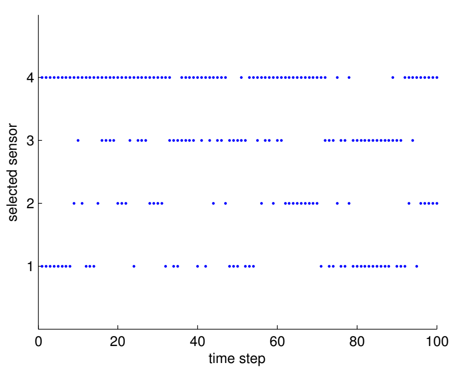


Fig. 6: Example schedule. A dot at time step k and sensor i means that sensor i was turned on at time step k.

0 and 2π, we modify the UKF by repositioning the sample points accordingly.

We consider four sensors with positions

$$\underline{s}_1 = \begin{pmatrix} 1 \\ 1.1 \end{pmatrix}, \ \underline{s}_2 = \begin{pmatrix} -1 \\ 1.1 \end{pmatrix}, \ \underline{s}_3 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \ \underline{s}_4 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} .$$

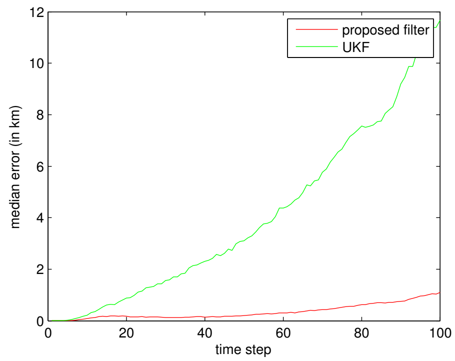
To model the behavior of the system, a constant velocity model with system noise $\mathbf{R} = \mathrm{diag}(0.001^2, 0.001^2, 0.001^2, 0.001^2)$ is used. The true initial state is given by $(-0.7, 0, 0.01, 0)^T$. This yields trajectories as depicted in Fig. 5.

The mean of the initial estimate is equal to the true initial state an the initial covariance is equal to the covariance of the system noise. The measurement uncertainty is given by $\sigma_i = 2$ for all $i = 1, \ldots, 4$. When such uncertainties are involved, approaches considering the circular nature of angular data promise better results than filters relying on the Gaussian distribution. We use the same scheduler with a planning horizon of two time steps for both filters. A schedule could look like the example in Fig. 6.

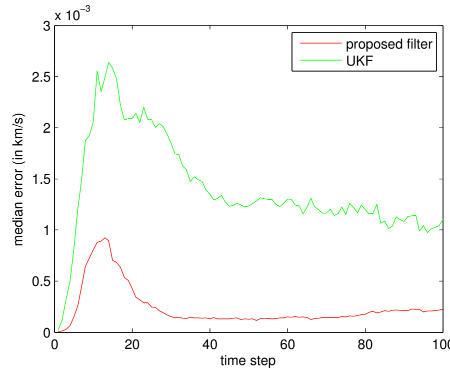
We performed 100 Monte Carlo runs to evaluate our filter. The root mean square error (RMSE) of the estimated position and velocity is given in Fig. 7. The lower and upper edges of each box are located at the first and third quartile of the simulation outcomes. It is obvious from these boxplots that our filter outperforms the UKF. If we plot the error over time, we can see that the UKF displays a bad performance throughout the simulation (Fig. 8 and Fig. 9).

V. Conclusions

In this paper, we have presented a novel algorithm for handling bearings-only measurements based on circular statistics. A deterministic sampling scheme for the wrapped normal



(a) Position.



(b) Velocity.

Fig. 9: Median of the RMSE.

distribution was derived. We showed how this scheme can be applied to estimate the position and velocity of a moving object based on scheduled bearings-only measurements. Furthermore, we have presented a sensor scheduling algorithm, which minimizes the estimation uncertainty while reducing the energy consumption of the sensors.

In simulations, we have demonstrated the viability of our approach. A comparison with the UKF shows that the proposed methods is superior when the measurement uncertainties are large.

Future work may include the extension to a deterministic sampling of the wrapped normal distribution with a larger number of Dirac components.

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