Decentralized Control Based on Globally Optimal Estimation

Marc Reinhardt, Benjamin Noack, and Uwe D. Hanebeck

Abstract— **A new method for globally optimal estimation in decentralized sensor-networks is applied to the decentralized control problem. The resulting approach is proven to be optimal when the nodes have access to all information in the network. More precisely, we utilize an algorithm for optimal distributed estimation in order to obtain local estimates whose combination yields the globally optimal estimate. When the interconnectivity is high, the local estimates are almost optimal, which motivates the application of the principle of separation. Thus, we optimize the controller and finally obtain a flexible algorithm, whose quality is evaluated in different scenarios. In applications where the strong requirements on a perfect communication cannot be guaranteed, we derive quality bounds by help of a detailed evaluation of the algorithm. When information is regularly exchanged, it is demonstrated that the algorithm performs almost optimally and therefore, offers system designers a flexible and easy to implement approach. The field of applications lies within the area of strongly networked systems, in particular, when communication disturbances cannot be foreseen or when the network structure is too complicated to apply optimized regulators.**

I. INTRODUCTION

Spatially distributed sensor-actuator-networks and large-scale systems require a decomposition of a central estimator and controller into small, independent sub-nodes that are somehow interconnected. Examples of such scenarios are electricity distribution grids, computer network systems, manufacturing systems etc.

In classical control theory, the controllers are required to have access to all information. In the scenarios described above, however, the local sensor nodes possess different information sets about the system that must be utilized locally – preferably in an optimal manner. Such systems are investigated in the field of decentralized control.

In the 60's and 70's, strong effort has been brought to the investigations of decentralized control [1]. Fundamental investigations about the optimality of linear controllers without feedback have been made [2]. Another major contribution was Witsenhausen's counterexample [3], stating that the optimal decentralized control law is nonlinear even in simple linear-quadratic-Gaussian (LQG) scenarios. In [4], the class of *partially nested* LQG problems has been identified, where partially nested structures have the characteristic that node A has access to all information of B, whenever a node A is affected by an operation of a node B. It has been shown that problems of this type can be solved optimally by decentralized linear controllers. For the common case, it has been shown that finding the LQG controller under decentralized information constraints is in general intractable [5], [6].

Besides these theoretical aspects, Chong [7] derived the optimal linear control strategy when no communication is established between the different nodes. Special classes such as the quadratic invariant problems [8] have been identified that allow a convex optimization of the controller design [8], [9]. Another approach is to identify which decentralized control problems have simple control sets and bound the possible solutions by help of Groebner bases [10].

Suboptimal approaches that allow a flexible communication have been presented in [11], [12], for example. Besides this direct optimization of linear models in order to minimize a cost function, there is an ongoing research utilizing graph theory [13], ideas of distributed artificial intelligence [14], and many others.

Recently, in estimation theory the Distributed Kalman Filter (DKF) has been proposed [15], [16] that allows to recover the globally optimal estimate from the distributed estimates when the utilized measurement models are known to all nodes. An extension of this algorithm, i.e., the Hypothesizing Distributed Kalman Filter (HKF) [17], [18], relaxes the hard constraint regarding the global knowledge about the measurement models while the optimality is maintained under specific conditions.

In this paper, we aim at bringing the results of linear estimation theory to the problem of linear decentralized control. We make use of the special structure of the estimates within the DKF, respectively the HKF, and propose, based on this, a flexible decentralized LQG algorithm. It is shown that the algorithm achieves globally optimal results when the sensors and actuators are completely cross-linked. Additionally, we investigate the performance of a sensor-actuator-network when no communication is allowed, but the decentralized control actions affect the system additively, and demonstrate that the new algorithm performs globally optimal when the feedback costs sum up.

The application areas of the proposed algorithm are wide as there are no requirements concerning the communication structure or the distribution of actuators and sensors. However, the application of the algorithm is, in particular, meaningful in scenarios where a full interconnectivity between the nodes in the network is employed, but, e.g., due to (seldom and unpredictable) packet-losses, the estimates of remote nodes do not arrive or arrive lately sometimes. As the local actuators must be capable of handling such packet-losses and the cost function of the control system should be almost optimal, it is neither meaningful to employ a central Kalman filter strategy nor a decentralized estimator approach that yields suboptimal

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results in general. We demonstrate that by utilizing the new approach, the costs are minimized when no packets are lost and are only slightly higher than in the central LQG controller otherwise.

We begin with the problem formulation and present the HKF as a new extension of the DKF. Based on this algorithm, we derive the new approach and demonstrate its effectiveness in several scenarios. Possible extensions and improvements are finally discussed in the conclusion.

II. Problem Statement

Let the number of actuators be M and the number of sensors be N. We consider a discrete-time, linear system model

$$\underline{\boldsymbol{x}}_{k+1} = \mathbf{A}_k \underline{\boldsymbol{x}}_k + \mathbf{B}_k \underline{u}_k + \underline{\boldsymbol{w}}_k \ , \tag{1}$$

where $\underline{\boldsymbol{x}} \in \mathbb{R}^n$, $\mathbf{B}_k = [\mathbf{B}_k^1, \ldots, \mathbf{B}_k^M]$, $\underline{\boldsymbol{w}}_k \sim \mathcal{N}(\underline{0}, \mathbf{C}_k^w)$, and $\underline{u}_k = \left[\left(\underline{u}_k^1\right)^\top, \ldots, \left(\underline{u}_k^M\right)^\top \right]^\top$. Let $\operatorname{diag}(\mathbf{G}_1, \ldots, \mathbf{G}_N)$ denote a block-diagonal matrix. The linear measurement model is given as

$$\underline{\boldsymbol{z}}_k = \mathbf{H}_k \underline{\boldsymbol{x}}_k + \underline{\boldsymbol{v}}_k \tag{2}$$

with $\mathbf{H}_k = \operatorname{diag}(\mathbf{H}_k^1, \ldots, \mathbf{H}_k^N)$ and white Gaussian noise $\underline{\boldsymbol{v}}_k \sim \mathcal{N}(\underline{0}, \mathbf{C}_k^v)$, where $\mathbf{C}_k^v = \operatorname{diag}(\mathbf{C}_k^{v_1}, \ldots, \mathbf{C}_k^{v_N})$. The dimension of the measurements $\underline{\boldsymbol{z}}_k = \left[\left(\underline{\boldsymbol{z}}_k^1\right)^\top, \ldots, \left(\underline{\boldsymbol{z}}_k^N\right)^\top \right]^\top$ fits to the dimension of the matrix-blocks in $\mathbf{C}_k^v$, $\mathbf{H}_k$ respectively. When all information is available at one node, the well-known Kalman filter can be utilized to combine the estimate $\hat{\underline{x}}_k$ with the measurement vector $\underline{\boldsymbol{z}}_k$ in order to obtain the minimum-mean-squared-error estimation.

However, in decentralized control, multiple sensors estimate the same state $\underline{\boldsymbol{x}}_k$. The distributed estimates $\left(\hat{\underline{x}}_k^1\right)^\top, \ldots, \left(\hat{\underline{x}}_k^N\right)^\top$ utilize the same system model but filter different measurements. Due to the common process noise from equation (1) and the globally optimal (Kalman) filter matrix, which is in general not block-diagonal, these estimates are usually correlated, which makes decentralized processing difficult. To emphasize this, the globally optimal distributed estimation over multiple time steps is no trivial problem that has been solved only under tight constraints regarding the local availability of knowledge about the utilized measurement models in the remote nodes. In particular, there is no estimation technique known that maintains the local optimality and the optimality of the estimate that is gained from the combination of the local estimates.

Let $\hat{\underline{x}}_k$ be the concatenated vector of the local estimates. The linear solution of the central time-variant LQG control problem is given by the feedback matrices $\mathbf{F}_k$ with $\underline{u}_k = -\mathbf{F}_k \hat{\underline{x}}_k$ and filter gains that minimize the cost function

$$J_T = \mathrm{E}\left\{ \left(\underline{\boldsymbol{x}}_T\right)^\top \mathbf{Q}_T \underline{\boldsymbol{x}}_T + \sum_{k=1}^{T-1} \left(\underline{\boldsymbol{x}}_k\right)^\top \mathbf{Q}_k \underline{\boldsymbol{x}}_k + \left(\underline{u}_k\right)^\top \mathbf{R}_k \underline{u}_k \right\} \tag{3}$$

with positive semi-definite, symmetric matrices $\mathbf{Q}_k$, $\forall k \in 1, \ldots, T$ and positive-definite, symmetric matrices $\mathbf{R}_k$, $\forall k \in 1, \ldots, T-1$. Depending on $T < \infty$, the controller that optimizes equation (3) is called the finite- or infinite-horizon, discrete-time LQG controller. Note that for infinite-horizon controllers, the respective matrices are assumed to be time-invariant. For both types, the LQG controller can be obtained in closed form by help of the corresponding Algebraic Riccati Equations (ARE).

In the remainder of this paper, we focus on finite-horizon controllers. Nevertheless, the considerations made are also true for infinite-horizon controllers and can be easily applied to them. We assume $T < \infty$ and obtain the finite-horizon ARE

$$\mathbf{P}_{k-1} = \mathbf{Q}_k + \mathbf{A}_k^\top \mathbf{P}_k \Big(\mathbf{I} - \mathbf{B}_k \left(\mathbf{R}_k + \mathbf{B}_k^\top \mathbf{P}_k \mathbf{B}_k \right)^{-1} \mathbf{B}_k^\top \mathbf{P}_k \Big) \mathbf{A}_k \tag{4}$$

with feedback matrices

$$\mathbf{F}_k = \left(\mathbf{R}_k + \mathbf{B}_k^\top \mathbf{P}_k \mathbf{B}_k \right)^{-1} \mathbf{B}_k^\top \mathbf{P}_k \mathbf{A}_k \ . \tag{5}$$

In order to derive the optimal decentralized regulator, one would optimize the cost function (3) with $\mathbf{R}_k = \operatorname{diag}(\mathbf{R}_k^1, \ldots, \mathbf{R}_k^M)$ with the secondary condition that $\mathbf{F}_k$ as well as the filter gain must be block diagonal matrices. Only when these two requirements are fulfilled, distributed processing is possible as otherwise the estimates of the distributed nodes would require the concrete instances of estimates or measurements of the other nodes to be available locally.

A minimization of this linear decentralized control problem is hard to derive [6] as the principle of separation does not hold in general for decentralized estimation [7]. This is due to the fact that in contrast to the central Kalman filter, the local estimates are in general suboptimal from a global point of view.

III. Globally Optimal Distributed Estimation

In the following, we present the DKF [15], [16], which allows the reconstruction of the globally optimal estimate and is the basis for the proposed decentralized control algorithm. In this paper, we follow the notation and formulation of [18]. In contrast to the original formulation, all assumptions about the estimation quality of the sensor network are bundled within one variable and a clear distinction between auxiliary and estimation variables is made.

Let the estimate of the i-th sensor node consists of mean vector $\hat{\underline{x}}_k^i$ and covariance matrix $\mathbf{C}_k^{x_i}$. Instead of a direct processing of the estimates, we introduce auxiliary variables

$$\underline{x}_k^i = \mathbf{C}_k^x \left(\hat{\mathbf{C}}_k^{x_i} \right)^{-1} \hat{\underline{x}}_k^i \ \text{with} \ \left(\mathbf{C}_k^x\right)^{-1} = \sum_{i=1}^{N} \left(\hat{\mathbf{C}}_k^{x_i} \right)^{-1} \ , \tag{6}$$

and perform the prediction and filter operations on these variables. More specifically, we apply a relaxed prediction model

$$\begin{aligned} \underline{x}_{k+1|k}^i &= \mathbf{A}_k \underline{x}_k^i \ , \\ \mathbf{C}_{k+1|k}^x &= \mathbf{A}_k \mathbf{C}_k^x \mathbf{A}_k^\top + \mathbf{C}_k^w \end{aligned} \tag{7}$$

and combine the estimates and measurements with gains

$$\mathbf{K}_k = \mathbf{C}_{k|k}^x \left(\mathbf{C}_{k|k-1}^x \right)^{-1} \ \text{and} \ \mathbf{L}_k^i = \mathbf{C}_{k|k}^x \left(\mathbf{H}_k^i \right)^\top \left(\mathbf{C}_k^{v_i} \right)^{-1} \ , \tag{8}$$

where the auxiliary covariance matrix $\mathbf{C}^x_{k|k}$ depends on a global measurement model

$$(\mathbf{P}^z_k)^{-1} = \sum_{i=1}^{N} \mathbf{H}^i_k \left(\mathbf{C}^{v_i}_k\right)^{-1} \left(\mathbf{H}^i_k\right)^\top \tag{9}$$

and is obtained by

$$\mathbf{C}^x_{k|k} = \left(\left(\mathbf{C}^x_{k|k-1}\right)^{-1} + \left(\mathbf{P}^z_k\right)^{-1}\right)^{-1} .$$

It is worth mentioning that the combination of measurement and estimate according to

$$\underline{x}^i_{k|k} = \mathbf{K}_k \ \underline{x}^i_{k|k-1} + \mathbf{L}^i_k \ \underline{\tilde{z}}^i_k \tag{10}$$

is not unbiased and thus, the auxiliary variable $\underline{x}^i_k$ is biased in general. Furthermore, note that the DKF can only be applied when the covariance matrices of all estimates are known to all nodes at the initialization step (6) and the utilized measurement models are known to all nodes at the filtering step (10).

However, when this knowledge is available and the auxiliary variables $\underline{x}^i_k$ are combined according to

$$\hat{\underline{x}}^f_k = \sum_{i=1}^{N} \underline{x}^i_k \ , \tag{11}$$

the estimate $\hat{\underline{x}}^f_k$ is unbiased and equals the globally optimal one.

As in many applications it is hard to guarantee that all sensors are functioning and that all measurement models are known to all nodes, we present the HKF [17], [18] as an extension to the standard DKF that relaxes these assumptions. While in the DKF it is inadvertently to know the measurement models of all nodes in the network, this is relaxed in the HKF by providing an assumption for $(\mathbf{P}^z_k)^{-1}$ instead of the exact value. When the sum of the uncertainty terms of the actually utilized models and $(\mathbf{P}^z_k)^{-1}$ are the same, i.e., (9) holds, the estimation result is globally optimal. Otherwise, the HKF still provides meaningful results where the estimation quality depends on the difference between the assumption about the global measurement model and the actually utilized one.

The basic idea of the HKF is to calculate a correction matrix $\mathbf{\Delta}^{x_i}_k$ that allows to derive an unbiased estimate from the auxiliary variable $\underline{x}^i_k$ by a multiplicative correction

$$\hat{\underline{x}}^i_k = \left(\mathbf{\Delta}^{x_i}_k\right)^{-1} \underline{x}^i_k \ . \tag{12}$$

The calculation of the inverse of this correction matrix is performed recursively. In order to derive the unbiased initial estimate, we invert the globalization transformation from (6) with $\mathbf{\Delta}^{x_i}_1 = \mathbf{C}^x_1(\hat{\mathbf{C}}^{x_i}_1)^{-1}$, $\forall i \in \{1,\ldots,N\}$. The bias elimination of the predicted estimate is led back to the bias elimination of the filtering step by defining

$$\mathbf{\Delta}^{x_i}_{k+1|k} = \mathbf{A}_k \mathbf{\Delta}^{x_i}_k \left(\mathbf{A}_k\right)^{-1} \ . \tag{13}$$

In the filtering step, we basically replace the auxiliary covariance matrix $\mathbf{C}^x_k$ by a new matrix that incorporates the actually utilized models instead of the assumed ones and set

$$\mathbf{\Delta}^{x_i}_{k|k} = \mathbf{C}^x_k \left(\mathbf{C}^{\Delta_i}_k\right)^{-1} \ , \tag{14}$$

where the new matrix is given by

$$\left(\mathbf{C}^{\Delta_i}_k\right)^{-1} = \left(\mathbf{C}^x_{k|k-1}\right)^{-1} \mathbf{\Delta}^{x_i}_{k|k-1} + \mathbf{H}^i_k \left(\mathbf{C}^{v_i}_k\right)^{-1} \left(\mathbf{H}^i_k\right)^\top .$$

Besides the presented local processing of estimates, an algorithm to combine multiple estimates has also been derived in [18]. W.l.o.g., let the variables $\underline{x}^1_k, \ldots, \underline{x}^{N_f}_k$ and the correction matrices $\mathbf{\Delta}^{x_1}_k, \ldots, \mathbf{\Delta}^{x_{N_f}}_k$ with $N_f \leq N$ be available at a fusion node. It has been shown that the variables

$$\underline{x}^f_k = \sum_{i=1}^{N_f} \underline{x}^i_k \ \text{and} \ \mathbf{\Delta}^{x_f}_k = \sum_{i=1}^{N_f} \mathbf{\Delta}^{x_i}_k \tag{15}$$

are identical to those variables that have processed all measurements comprised in $\underline{x}^i_k$, $\forall i \in \{1,\ldots,N_f\}$ centrally at one node with the HKF.

Therefore, the HKF provides the procedures to eliminate the bias from auxiliary variables that have been calculated with the DKF. Some attributes of the estimates that are obtained by (12) have been derived in [17] and [18] including the conditions for the global optimality of the estimates and an evaluation of the estimation performance when the assumption about the measurement models does not meet the actually utilized one.

In summary, the HKF is a distributed estimation algorithm that guarantees globally optimal estimation results when the overall measurement quality of the network is known to all nodes and still provides meaningful, but slightly suboptimal results otherwise.

IV. DEZENTRALIZED LQR

In this section, we apply the HKF to the decentralized control problem. The basic idea of this approach is to fully exploit the distributed estimation information in highly connected sensor-actuator-networks and to employ controllers that are optimized by help of the central ARE. As the estimates at the different nodes are almost equal to the optimal central estimate when the communication rate is high, the control actions are likewise similar to the optimal ones, which is also demonstrated in the evaluation section later on.

When the globally optimal estimate is calculated, the estimate is the conditional mean value $\hat{\underline{x}}_k = \mathrm{E}\{\underline{\boldsymbol{x}}|\mathbf{Z}_k,\ldots,\mathbf{Z}_1\} \in \mathbb{R}^n$ with $\mathbf{Z}_t = \{\underline{z}^1_t,\ldots,\underline{z}^N_t\}$, $\forall t \in 1,\ldots,k$. In this case, the optimization of the cost function (3) yields the discrete-time, finite-horizon ARE (4).

The optimal feedback control matrix is then given as

$$\mathbf{F}_k = \left(\mathbf{R}_k + \mathbf{B}^\top_k \mathbf{P}_k \mathbf{B}_k\right)^{-1} \mathbf{B}^\top_k \mathbf{P}_k \mathbf{A}_k \ . \tag{16}$$

Applying the matrix inversion lemma, it can be shown that the following equality holds

$$\left(\mathbf{R}_k + \mathbf{B}^\top_k \mathbf{P}_k \mathbf{B}_k\right)^{-1} = \mathbf{R}^{-1}_k \Big(\mathbf{I} - \mathbf{B}^\top_k \left(\mathbf{P}^{-1}_k + \mathbf{B}_k \mathbf{R}^{-1}_k \mathbf{B}^\top_k\right)^{-1} \mathbf{B}_k\Big) \mathbf{R}^{-1}_k \ . \tag{17}$$

We define $\mathbf{D}_k = \left(\mathbf{P}_k^{-1} + \mathbf{B}_k \mathbf{R}_k^{-1} \mathbf{B}_k^\top\right)^{-1}$ and obtain

$$\left(\mathbf{R}_k + \mathbf{B}_k^\top \mathbf{P}_k \mathbf{B}_k\right)^{-1} = \begin{pmatrix} \mathbf{E}_k^{11} & \dots & \mathbf{E}_k^{1M} \\ \vdots & & \vdots \\ \mathbf{E}_k^{M1} & \dots & \mathbf{E}_k^{MM} \end{pmatrix} \quad (18)$$

with $\mathbf{E}_k^{ij} = \left(\mathbf{R}_k^i\right)^{-1}\left(\mathbf{I} - \left(\mathbf{B}_k^i\right)^\top \mathbf{D}_k \mathbf{B}_k^j\right)\left(\mathbf{R}_k^j\right)^{-1}$. The optimal central feedback matrix $\mathbf{F}_k$ is therefore given as

$$\mathbf{F}_k = \begin{pmatrix} \mathbf{F}_k^1 \\ \vdots \\ \mathbf{F}_k^M \end{pmatrix} = \begin{pmatrix} \left(\sum_{j=1}^M \mathbf{E}_k^{1j} \left(\mathbf{B}_k^j\right)^\top\right) \mathbf{P}_k \mathbf{A}_k \\ \vdots \\ \left(\sum_{j=1}^M \mathbf{E}_k^{Mj} \left(\mathbf{B}_k^j\right)^\top\right) \mathbf{P}_k \mathbf{A}_k \end{pmatrix} . \quad (19)$$

The control input vector $\underline{u}_k = \mathbf{F}_k \hat{\underline{x}}_k$ can be denoted as

$$\begin{pmatrix} \underline{u}_k^1 \\ \vdots \\ \underline{u}_k^M \end{pmatrix} = - \begin{pmatrix} \mathbf{F}_k^1 \\ \vdots \\ \mathbf{F}_k^M \end{pmatrix} \hat{\underline{x}}_k \ , \quad (20)$$

where the sub-vectors $\underline{u}_k^i, \forall i \in \{1, \dots, M\}$ can be interpreted as local control inputs.

As we utilize the HKF algorithm, we can guarantee that the local estimates equal the conditional mean $\hat{\underline{x}}_k$ when all information about the measurements has been exchanged between the nodes. Otherwise, when the full interconnectivity cannot be guaranteed, it still holds

$$\mathrm{E}\{\underline{\boldsymbol{x}}_k\} = \mathrm{E}\{\hat{\underline{x}}_k\} = \mathrm{E}\left\{\left(\boldsymbol{\Delta}_k^L\right)^{-1} \sum_{i=1}^L \underline{x}_k^i\right\} \ , \quad (21)$$

and therefore, the control is still meaningful. Additionally, as we optimize the estimation quality and expect the information exchange between the sensor nodes to be high, the uncertainty of the estimates is low in general, which in combination with the unbiasedness (21) leads to estimates that are approximately equal. However, when the estimates are equal, they correspond to the globally optimal estimation[1] and therefore, the optimal LQR is given as (16).

This allows us to obtain the control inputs $\underline{u}_k^i, \ \forall i \in 1, \dots, M$ similar to (20) in a decentralized manner by calculating

$$\begin{pmatrix} \underline{u}_k^1 \\ \vdots \\ \underline{u}_k^M \end{pmatrix} = - \begin{pmatrix} \mathbf{F}_k^1 \left(\boldsymbol{\Delta}_k^{x_{f_1}}\right)^{-1} \underline{x}_k^{f_1} \\ \vdots \\ \mathbf{F}_k^M \left(\boldsymbol{\Delta}_k^{x_{f_M}}\right)^{-1} \underline{x}_k^{f_M} \end{pmatrix} ,$$

where $\boldsymbol{\Delta}_k^{x_{f_i}}$ denotes the fused correction matrix from (15) and $\underline{x}_k^{f_i}$ the corresponding local estimate that incorporates all transmitted estimates. When the nodes are completely interconnected and no packet-loss occurs, it holds $\boldsymbol{\Delta}_k^{x_{f_i}} = \mathbf{I}$ and $\underline{x}_k^{f_i} = \hat{\underline{x}}_k$.

In summary, the algorithm consists of two parts. The first one is decentralized estimation that is realized with the HKF. At each sensor node, a synthetic variable $\underline{x}_k^i$ and the corresponding correction matrix $\boldsymbol{\Delta}_k^{x_i}$ is maintained and later on exchanged, e.g., by a broadcast in wireless sensor networks, in order to allow each actuator node to reconstruct an unbiased estimate. As it has been shown, these estimates are optimal when a full rate communication is employed and so, we calculate the controller actions based on the central optimal ARE.

Note, that the proposed algorithm is suboptimal when not all information is exchanged and the local estimates differ from each other. In this case, the principle of separation does not hold and a cost function optimization, considering the actual communication, would yield other filter gains and feedback matrices. However, such an approach requires to solve an optimization problem and to communicate its solution to the nodes, which is especially hard when the communication is unreliable. Apart from that, the estimation quality of the proposed algorithm is almost optimal as it is shown in the next section and so, the advantage of optimizing the gains is negligible when the communication exchange is high.

[1]The combination of the local estimates yields the globally optimal results and when the local estimates are equal, their combination is the same vector as the local estimates.

V. Evaluation

Before the proposed algorithm is evaluated in detail, we present an example in order to prove its effectiveness, even if no communication is employed.

Example V.1 From the attributes of the DKF, we know that the sum of the local estimates matches $\hat{\underline{x}}_k$. This equality becomes valuable, when $\mathbf{B}_k^i = \mathbf{B}_k^j$ and the decentralized control costs are summing up. Assume a network consisting of nodes that are actuators and sensors at once to be given. Due to economic or other reasons, communication between the nodes cannot be employed. The idea is to split up the global estimation into a sum of local estimates.

Assume the local variables $\underline{x}_k^i$ to be globalized as it follows from (6) so that

$$\hat{\underline{x}}_k^f = \sum_{i=1}^N \underline{x}_k^i \quad (22)$$

holds. The representation of the central estimate as a sum allows to employ the central LQG control by simply mapping the state estimates linearly to the control actions. The optimal feedback matrix $\mathbf{F}_k$ is given by the LQR and thus, the control value $\underline{u}_k$ is obtained by

$$\underline{u}_k = -\mathbf{F}_k \hat{\underline{x}}_k^f \ . \quad (23)$$

With (22), this is split up into

$$\underline{u}_k = -\mathbf{F}_k \left(\underline{x}_k^1 + \dots + \underline{x}_k^N\right) \ . \quad (24)$$

When we simply map the globalized variables $\underline{x}_k^i$ to one addend of $\underline{u}_k$ each, we obtain

$$\underline{u}_k^i = -\mathbf{F}_k \underline{x}_k^i \text{ with } \underline{u}_k = \sum_{i=1}^N \underline{u}_k^i \ , \quad (25)$$

which allows us to calculate the optimal control action from the locally available variables $\underline{x}_k^i$ without any communication.

Note that the control inputs $\underline{u}_k^i$ differ in general, as the globalized variables $\underline{x}_k^i$ are biased when the measurement models are different. Thus, the costs are only optimal when not the costs for the local control actions, but only their sum is relevant. However, for applications such as the decentralized overpressure-control with multiple electronic valves where only the overall gas-pressure as well as the total amount of discharged gas is relevant, the algorithm is globally optimal.

For the detailed evaluation, assume the system model to describe an object that moves with constant velocity. The aim is to control the object decentralized in order to minimize the state variable that consists of position and velocity. We are given ten sensor-actuator-nodes of different types. More precisely, the input matrices and the measurement uncertainties of five small nodes are given by

i	$\mathbf{B}^i$	$\mathbf{C}^{v_i}$
1	$\operatorname{diag}(10, 0)$	$\operatorname{diag}(30, 20)$
2	$\operatorname{diag}(1, 0)$	$\operatorname{diag}(30, 20)$
3	$\operatorname{diag}(1, 1)$	$\operatorname{diag}(30, 20)$
4	$\operatorname{diag}(0, 1)$	$\operatorname{diag}(30, 20)$
5	$\operatorname{diag}(0, 10)$	$\operatorname{diag}(30, 20)$

.

The other five nodes are of the same type, but are equipped with sensors that obtain measurements with half the uncertainty compared to the small nodes. The system model is given as

$$\underline{\boldsymbol{x}}_{k+1} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \underline{\boldsymbol{x}}_k + \mathbf{B}\, \underline{u}_k + \underline{\boldsymbol{w}}_k \tag{26}$$

with $\underline{\boldsymbol{w}}_k \sim \mathcal{N}(\underline{0}, \operatorname{diag}(10, 20))$ and $\underline{\boldsymbol{x}}_1 \sim \mathcal{N}(\underline{0}, \operatorname{diag}(10, 10))$. The measurement matrices are $\mathbf{H}_k^i = \mathbf{I}$, $\forall i, k$ and the state cost matrix is $\mathbf{Q} = \operatorname{diag}(0.1, 0.05)$.

Each node estimates the state locally by making use of the HKF. The performance of the algorithm is measured over 15 time steps in 1000 Monte Carlo runs for different control cost matrices $\mathbf{R}$ and different communication rates between the nodes. For simplicity, the estimates of other nodes are used to find optimized control inputs only, but are not utilized to improve the local estimate.

As can be seen in Figure 1, the costs are almost optimal as long as the communication rate is above 60 percent. This result is independent of the relation between state and control costs. However, if the communication rate drops significantly, the control costs are remarkably higher than the costs of the optimal central LQG controller as the expected costs of the best linear decentralized controller are higher than those of the central approach and secondly, the assumption about the measurement models no longer holds, which makes the decentralized application of the central LQR suboptimal.

In a second scenario, each node has been cloned and the process noise uncertainty is modified in order to evaluate the performance of the proposed approach for different combinations of measurement and process noise. We set $\mathbf{R} = \operatorname{diag}(1, 0.5)$ and investigate the average costs in 1000 Monte Carlo runs for multiples of $\mathbf{C}^w = \operatorname{diag}(10, 20)$.

As it becomes clear from Figure 2, the performance of the algorithm suffers from packet-losses, in particular, when the process noise is low. For a process noise $0.14 \cdot \mathbf{C}^w$ the relative cost increments compared to the central LQG control law are given for the different communication rates (cr) by:

100_{cr}	80_{cr}	60_{cr}	40_{cr}	20_{cr}	0_{cr}
0%	2.23%	5.36%	11.87%	32.04%	146.47%

.

Thus, even when unrealistic worst-case scenarios are investigated, the proposed approach still works well for communication rates above 60%. Additional scenarios have been omitted for space reasons. Nevertheless, evaluations have shown that the performance is similar when other system and measurement models are utilized or the cost matrices are investigated for other relations between them.

It is worth pointing out, that the computational effort of the proposed algorithm in the local nodes is basically defined by the summation in (15) and the inversion in (12). The approximation of the control inputs of the other nodes can be simulated by utilizing the central feedback matrix. Although no computationally expensive nonlinear operations or optimizations have to be performed, the computational effort can be critical for small sensor-actuator-nodes. In scenarios where this is the case, the effort can be lowered by additional communication when only the own control input is calculated and is exchanged afterwards. When the communication fails, the missing control input can still be simulated afterwards. Such an approach does not only lower the computational effort but also improves the quality of the predicted estimate as the actually utilized control inputs are used instead of the estimated ones.

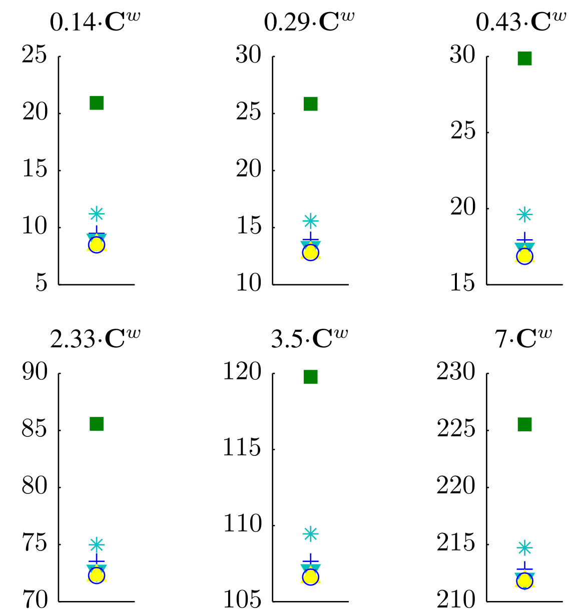

Fig. 2. The average costs over 15 time steps in 1000 Monte Carlo runs for different process noise uncertainties and for different communication rates between the nodes.

VI. Conclusion

In this paper, we proposed a new approach for decentralized control. The algorithm yields globally optimal results when a full information exchange between the sensors and actuators

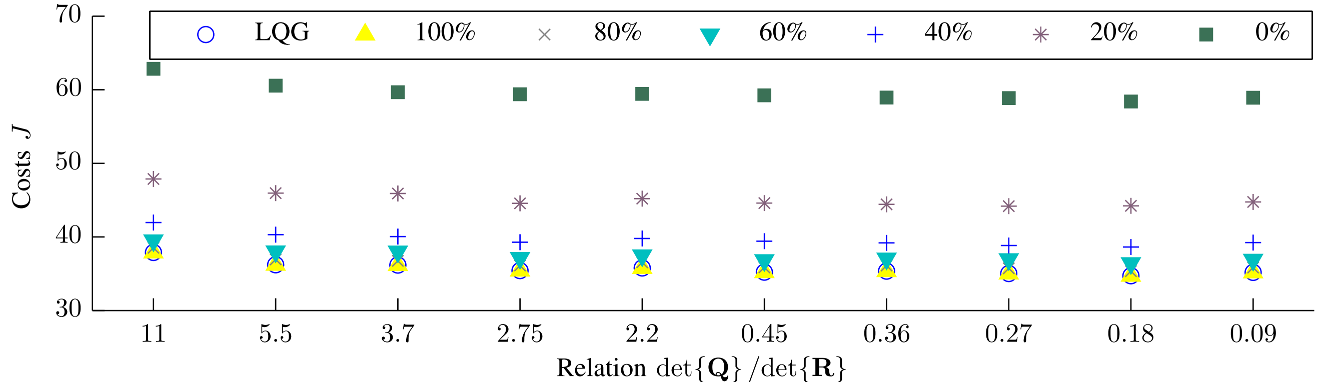

Fig. 1. The average costs over 15 time steps in 1000 Monte Carlo runs for different relations between state and control costs and for different communication rates between the nodes.

is employed. When this condition is not fulfilled, the approach is in general suboptimal, but still yields promising results as it has been shown by means of an evaluation in different scenarios. Fields of applications are, in particular, linear problems where an optimal decentralized solution cannot be obtained, the communication between the nodes cannot be foreseen, or when the design effort should be small.

The idea behind the proposed approach is to apply a new estimation algorithm to the decentralized control problem that allows the reconstruction of the globally optimal estimate from local estimates. As this estimation algorithm still provides almost optimal estimates even when the communication constraints are not fulfilled, we have shown that the application of the centrally optimal LQR is meaningful. We have demonstrated by means of an evaluation that the algorithm is robust towards packet-losses and provides approximately optimal results when the communication failure-rate is below 40%. Additionally, we have presented a problem class, in which the actuator impact is additive and the control costs sum up, where a special version of the proposed algorithm yields globally optimal results, even if no communication between the actuators is employed.

The proposed algorithm is only a starting point that still leaves room for improvement. Straightforward extensions are the communication of control inputs in order to improve the estimates of remote nodes as well as optimizations of the feedback matrix, other than the central optimization. For example when the rate of communication and thus, the uncertainty of the estimates at the different nodes can approximately be estimated, it may be meaningful to include this knowledge in the optimization of the feedback control matrix.

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References

[1] N. Sandell Jr, P. Varaiya, M. Athans, and M. Safonov, “Survey of Decentralized Control Methods for Large Scale Systems,” *IEEE Transactions on Automatic Control*, vol. 23, no. 2, pp. 108–128, 1978.

[2] R. Radner, “Team Decision Problems,” *The Annals of Mathematical Statistics*, vol. 33, no. 3, pp. 857–881, 1962.

[3] H. S. Witsenhausen, “A Counterexample in Stochastic Optimum Control,” *SIAM Journal on Control*, vol. 6, p. 131, 1968.

[4] Y. C. Ho and Others, “Team Decision Theory and Information Structures in Optimal Control Problems–Part I,” *IEEE Transactions on Automatic Control*, vol. 17, no. 1, pp. 15–22, 1972.

[5] V. Blondel and J. N. Tsitsiklis, “NP-Hardness of Some Linear Control Design Problems,” in *Proceedings of the 34th IEEE Conference on Decision and Control (CDC 1995)*, vol. 3. IEEE, 1995, pp. 2910–2915.

[6] V. D. Blondel and J. N. Tsitsiklis, “A Survey of Computational Complexity Results in Systems and Control,” *Automatica*, vol. 36, no. 9, pp. 1249–1274, 2000.

[7] C. Y. Chong and M. Athans, “On the Stochastic Control of Linear Systems with Different Information Sets,” *IEEE Transactions on Automatic Control*, vol. 16, no. 5, pp. 423–430, 1971.

[8] M. Rotkowitz and S. Lall, “Decentralized Control Information Structures Preserved under Feedback,” in *Proceedings of the 41st IEEE Conference on Decision and Control (CDC 2002)*, vol. 1. IEEE, 2002, pp. 569–575.

[9] J. H. Kim, S. Lall, W. Merrill, and A. Behbahani, “A Computational Approach for Decentralized Control of Turbine Engines,” in *Proceedings of the 49th IEEE Conference on Decision and Control (CDC 2010)*. IEEE, 2010, pp. 346–351.

[10] H. S. Shin and S. Lall, “Optimal Decentralized Control of Linear Systems via Groebner Bases and Variable Elimination,” in *Proceedings of the American Control Conference 2010*. IEEE, 2010, pp. 5608–5613.

[11] M. Aoki, “On Decentralized Linear Stochastic Control Problems with Quadratic Cost,” *IEEE Transactions on Automatic Control*, vol. 18, no. 3, pp. 243–250, 1973.

[12] J. Speyer, “Computation and Transmission Requirements for a Decentralized Linear-Quadratic-Gaussian Control Problem,” *IEEE Transactions on Automatic Control*, vol. 24, no. 2, pp. 266–269, 1979.

[13] G. Lafferriere, A. Williams, J. Caughman, and J. J. P. Veerman, “Decentralized Control of Vehicle Formations,” *Systems & Control Letters*, vol. 54, no. 9, pp. 899–910, 2005.

[14] M. R. Genesereth, J. S. Rosenschein, M. L. Ginsberg, S. U. D. of Computer Science, and S. U. H. P. Project, *Cooperation Without Communication*. Heuristic Programming Project, Computer Science Department, Stanford University, 1984.

[15] W. Koch, “On Optimal Distributed Kalman Filtering and Retrodiction at Arbitrary Communication Rates for Maneuvering Targets,” in *Proceedings of the 2008 IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems (MFI 2008)*, 2008, pp. 457–462.

[16] F. Govaers and W. Koch, “On the Globalized Likelihood Function for Exact Track-to-Track Fusion at Arbitrary Instants of Time,” in *Proceedings of the 14th International Conference on Information Fusion (FUSION 2011)*, 2011, pp. 1–5.

[17] M. Reinhardt, B. Noack, and U. D. Hanebeck, “On Optimal Distributed Kalman Filtering in Non-ideal Situations,” in *Proceedings of the 15th International Conference on Information Fusion (FUSION 2012)*, 2012.

[18] ——, “The Hypothesizing Distributed Kalman Filter,” in *Proceedings of the 2012 IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems (MFI 2012)*, 2012.