

# Passivity-based Dynamic Visual Feedback Control of Manipulators with Kinematic Redundancy

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**Abstract**—In this paper a passivity-based dynamic visual feedback control system, based on extended task space formulation, is addressed to control the kinematically redundant manipulator. Specifically, we consider the target tracking problem of dynamic visual feedback systems in the 3D-workspace. Firstly the brief summary of the visual feedback system is given with the fundamental representation of a relative rigid body motion. Secondly we derive the passivity of the dynamic visual feedback system by combining the manipulator dynamics (expressed in extended task space) and the visual feedback system. Using the proposed dynamic visual feedback control, one can optimize a performance index while tracking a moving target object. Finally we present simulation results to confirm the effectiveness of the proposed visual feedback control method and to verify the stability of both the end-effector motion and the null-space motion.

## I. INTRODUCTION

Robots and intelligent machines need large amounts of information to deal autonomously with objects in dynamical environments. Visual information has proven to be a highly effective means to recognize unknown surroundings. Vision-based control of robotic systems is the fusion of results from robot kinematics, dynamics, and computer vision systems to control the position of the robot end-effector in an efficient manner. The combination of mechanical control with visual information, so-called *visual feedback control* or *visual servoing*, should become essential when we consider a mechanical system working in *dynamical* environments [1][2].

A common characteristic in classical visual servoing is that the manipulator dynamics is negligible and does not interact with the visual feedback loop. However, this assumption is invalid for high speed tasks, whereas it holds for kinematic control problems [3]. For this reason our previous work [4] has investigated and proposed a passivity-based visual feedback control which takes the dynamic of the manipulator into account. However, the proposed control law and its proof of passivity is only valid for non-redundant manipulators. In this paper we will generalize our earlier work by extending the control method for redundant manipulators.

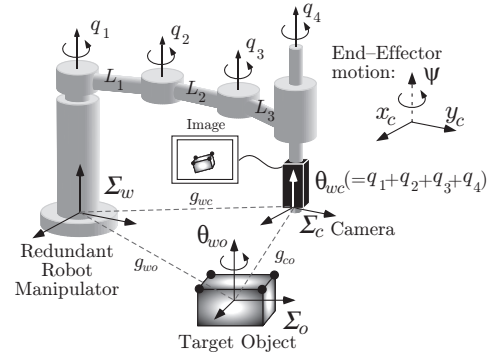


Fig. 1. Coordinate frames for a dynamic visual feedback system with redundant manipulator

A robot manipulator is defined as *redundant* if it possesses more degrees of freedom than are required to achieve the desired end-effector motion. The redundancy of such manipulators can be effectively used for the achievement of an additional performance such as: avoiding obstacles in the workspace and singular configuration, keeping within joint limits, avoiding instability for redundant manipulators mounted on a flexible base [5] or to optimize various performance criterion. If  $n$  is the degree of freedom of the manipulator and  $m$  is the degree of freedom necessary to perform a given task, then the degree of redundancy is given by  $r = n - m$ .

For the control of a redundant manipulator, many algorithms have been proposed based on an *extended task space*. In their work K. C. Park [6] and W. K. Chung [7] proposed an extended task space formulation for the control of both the end-effector motion and the null-space motion. The basic idea of the scheme is to augment the kinematic equation with the additional null-space kinematic equation. However, the control method in this form can not exactly be applied for visual feedback control.

It is well known that the technological fields, which need visual feedback control, are undoubtedly increasing; such as the laparoscopic surgery [8]. For such intelligent machines in future applications, kinematic redundancy will play an important role, e.g., for ensuring the safety in the laparoscopic surgery. Until now, there was no control method which considered the interaction of the visual feedback system with the redundant manipulator dynamics.

In this paper we will consider the control and the estimation problems for dynamic visual feedback control of *redundant manipulators*. Describing the dynamic visual

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feedback system in the so-called *extended task space*, this framework will generalize our previous works a great deal. An important feature of this paper is the investigation and proof of a passivity-based dynamic visual feedback control law expressed in extended task space. Furthermore, we will demonstrate the effectiveness of the proposed control method by means of several simulations.

Throughout this paper, we use the notation  $e^{\hat{\xi}\theta_{ab}} \in \mathbb{R}^{3 \times 3}$  to represent the change of the principle axes of a frame  $\Sigma_b$  relative to a frame  $\Sigma_a$ .  $\xi_{ab} \in \mathbb{R}^3$  specifies the direction of rotation and  $\theta_{ab} \in \mathbb{R}$  is the angle of rotation. The notation ' $\wedge$ ' (wedge) is the skew-symmetric operator such that  $\hat{\xi}\theta = \xi \times \theta$  for the vector cross-product  $\times$  and any vector  $\theta \in \mathbb{R}^3$ . The notation ' $\vee$ ' (vee) denotes the inverse operator to ' $\wedge$ '. We use the  $4 \times 4$  matrix

$$g_{ab} = \begin{bmatrix} e^{\hat{\xi}\theta_{ab}} & p_{ab} \\ 0 & 1 \end{bmatrix} \quad (1)$$

as the homogeneous representation of  $g_{ab} = (p_{ab}, e^{\hat{\xi}\theta_{ab}})$ , which is the description of the configuration of a frame  $\Sigma_b$  relative to a frame  $\Sigma_a$ .

The adjoint transformation associated with  $g_{ab}$ , written  $\text{Ad}_{(g_{ab})}$ , is given by

$$\text{Ad}_{(g_{ab})} = \begin{bmatrix} e^{\hat{\xi}\theta_{ab}} & \hat{p}_{ab}e^{\hat{\xi}\theta_{ab}} \\ 0 & e^{\hat{\xi}\theta_{ab}} \end{bmatrix}. \quad (2)$$

For a more detailed description to the mathematical machinery used in this paper we refer to [9].

## II. VISUAL FEEDBACK SYSTEM

### A. Fundamental Representation for Visual Feedback System

In this paper we consider a visual feedback system which consists of three coordinate frames; depicted in Fig. 1. The coordinate frames  $\Sigma_w$ ,  $\Sigma_o$ , and  $\Sigma_c$  represent the world frame, the target object frame, and the camera (end-effector) frame, respectively. As it is illustrated in Fig. 1  $g_{wc}$ ,  $g_{wo}$ , and  $g_{co}$  represents the relative rigid body motion from  $\Sigma_w$  to  $\Sigma_c$ , from  $\Sigma_w$  to  $\Sigma_o$ , and from  $\Sigma_c$  to  $\Sigma_o$ , respectively.

In the last section it was indicated that for redundant manipulators it is possible to achieve various subtasks simultaneously. In the case of vision-based control, the tasks to be performed by redundant manipulators can be grouped into two classes.

The first objective is to control the end-effector motion. To be more precise, the objective is to bring the actual relative rigid body motion  $g_{co}$  to a given reference  $g_d$ . Our goal is to determine the robot motion via the visual information for this purpose. The reference  $g_d$  for the rigid body motion  $g_{co}$  is assumed to be constant throughout this paper, because the camera can track the moving target object in this case.

The secondary objective is to control the null-space motion in order to achieve a second performance, i.e. rearrangement of the structure of the manipulator without any influences on the end-effector motion.

The relative rigid body motion involves the velocity of both the camera and the moving target object. For this reason,

let us consider the velocity of each rigid body as described in [9]. The body velocity of the camera relative to the world frame  $\Sigma_w$  can be denoted as

$$\hat{V}_{wc}^b = g_{wc}^{-1} \dot{g}_{wc} = \begin{bmatrix} \hat{\omega}_{wc} & v_{wc} \\ 0 & 0 \end{bmatrix}, \quad V_{wc}^b = \begin{bmatrix} v_{wc} \\ \omega_{wc} \end{bmatrix}, \quad (3)$$

where  $v_{wc}$  and  $\omega_{wc}$  represent the velocity of the origin and the angular velocity from  $\Sigma_w$  to  $\Sigma_c$ , respectively ([9], Chap.2, eq.(2.55)).

Then, the fundamental representation of the relative rigid body motion  $g_{co}$  is described as follows [4]

$$V_{co}^b = -\text{Ad}_{(g_{co}^{-1})} V_{wc}^b + V_{wo}^b, \quad (4)$$

where  $V_{wo}^b$  is the body velocity of the target object relative to  $\Sigma_w$  and  $\text{Ad}_{(g_{ab})}$  is the adjoint transformation associated with  $g_{ab}$  [9]. Roughly speaking, the relative rigid body motion  $g_{co}$  will be derived from the difference between the camera velocity  $V_{wc}^b$  and the target object velocity  $V_{wo}^b$ .

Due to the fact that the target object velocity  $V_{wo}^b$  is unknown and furthermore can not be measured directly the relative rigid body motion  $g_{co}$  can not be immediately obtained. For this reason, we have to derive an *estimation error* and *control error system*, which estimates the relative rigid body motion from the image information and tracks the estimated motion of the moving target object.

### B. Estimation Error and Control Error Systems

Here, a brief summary of our prior works is given [4]. The visual information  $f$  which includes the relative rigid body motion can be exploited, while the relative rigid body motion  $g_{co}$  can not be obtained directly in the visual feedback system. In order to bring the actual relative rigid body motion  $g_{co}$  to a given reference  $g_d$  in Fig. 1, we consider the following model which just comes from the fundamental representation (4).

$$\bar{V}_{co}^b = -\text{Ad}_{(\bar{g}_{co}^{-1})} V_{wc}^b + u_e \quad (5)$$

where  $u_e$  is the input in order to converge the estimated value to the actual relative rigid body motion. Because of the design of  $u_e$  needs a property of the whole visual feedback system, we will propose  $u_e$  in Section III-C. For the remainder of this work, the bar denotes estimated values, e.g.,  $\bar{V}_{co}^b$  denotes the estimated value of the relative rigid body motion  $V_{co}^b$ .

Next, the estimation error of the relative rigid body motion from  $\Sigma_c$  to  $\Sigma_o$ , i.e. the error between  $\bar{g}_{co}$  and  $g_{co}$ , is defined as

$$g_{ee} = \bar{g}_{co}^{-1} g_{co} \quad (6)$$

which is called the estimated error. Using the notation  $e_R(e^{\hat{\xi}\theta_{ab}}) = \frac{1}{2}(e^{\hat{\xi}\theta_{ab}} - e^{-\hat{\xi}\theta_{ab}})$  [10], the vector of the estimated error is given by  $e_e := [p_{ee}^T, e_R^T(e^{\hat{\xi}\theta_{ee}})]^T$ . Then, the estimation error vector  $e_e$  can be obtained by using image information  $f$ . The estimated error system is represented by

$$V_{ee}^b = -\text{Ad}_{(g_{ee}^{-1})} u_e + V_{wo}^b. \quad (7)$$

Similarly, we define the error between  $g_d$  and  $\bar{g}_{co}$ , which is called the control error, as follows

$$g_{ec} = g_d^{-1} \bar{g}_{co}. \quad (8)$$

The vector of the control error is defined as  $e_c := [p_{ec}^T, e_R^T(e^{\hat{\xi}\theta_{ec}})]^T$ . The control error system is described by

$$V_{ec}^b = -\text{Ad}_{(\bar{g}_{co}^{-1})} V_{wc}^b + u_e, \quad (9)$$

Because of the design of  $u_e$  needs to be a property of the whole visual feedback system, we will propose  $u_e$  in Section III-C.

Combining (7) and (9), we construct the visual feedback system as follows

$$\begin{bmatrix} V_{ec}^b \\ V_{ee}^b \end{bmatrix} = \begin{bmatrix} -\text{Ad}_{(\bar{g}_{co}^{-1})} & I \\ 0 & -\text{Ad}_{(g_{ee}^{-1})} \end{bmatrix} u_{ce} + \begin{bmatrix} 0 \\ I \end{bmatrix} V_{wo}^b \quad (10)$$

where  $u_{ce} := [(V_{wc}^b)^T, u_e^T]$  denotes the control input. Let us define the error vector of the visual feedback system as  $e_{ec} := [e_c^T, e_e^T]^T$  which contains the control error vector  $e_c$  and the estimation error vector  $e_e$ . Here, we define the output of the visual feedback system (10) as

$$\nu_{ce} = \begin{bmatrix} -\text{Ad}_{(g_d^{-1})}^T & 0 \\ \text{Ad}_{(e^{-\hat{\xi}\theta_{ec}})} & -I \end{bmatrix} e_{ce}, \quad (11)$$

then the visual feedback system (10) satisfies  $\int_0^T u_{ce}^T \nu_{ce} d\tau \geq -\beta_{ce}$  where  $\beta_{ce}$  is a positive scalar. This would suggest that the visual feedback system (10) is *passive* from the input  $u_{ce}$  to the output  $\nu_{ce}$  just formally as in the definition in [11]. For a more detailed description we refer to our previous work [4].

### III. DYNAMIC VISUAL FEEDBACK CONTROL

#### A. Kinematic Expression for Redundant Manipulators

Since the camera is mounted on the end-effector of the manipulator, the body velocity of the camera  $V_{wc}^b$  is given by

$$V_{wc}^b = J_b(q) \dot{q}, \quad (12)$$

where  $J_b(q) \in \mathbb{R}^{m \times n}$  is the manipulator body Jacobian [9]. Although this equation describes the end-effector motion completely, in the case of redundant manipulator there exists an additional motion; the so-called null-space motion.

Now, let us define a matrix  $J_n(q) \in \mathbb{R}^{r \times n}$ , which consists of the basis vectors spanning the null space of  $J_b$ . There are a few methods available to obtain the null-space matrix  $J_n$  [6].

Apart from this, let us define  $\dot{x}_n$  as a self-motion velocity or null-space velocity. Then we have a *complementary mapping relationship* between the joint space and the null-space, which is

$$\dot{x}_n = J_n(q) \dot{q}. \quad (13)$$

This equation describes the null-space motion.

Combining (12) and (13) leads to an *extended task space*, as described in [6]

$$J_e = \begin{bmatrix} J_b \\ J_n \end{bmatrix}, \quad \dot{x}_e = \begin{bmatrix} V_{wc}^b \\ \dot{x}_n \end{bmatrix}. \quad (14)$$

Note that if the manipulator is not in singular configuration, the extended Jacobian matrix  $J_e(q) \in \mathbb{R}^{n \times n}$  is always a quadratic matrix and a full rank matrix, thus invertible. Now, the extended Jacobian equation is determined as

$$\dot{x}_e = J_e \dot{q} \quad \dot{q} = J_e^{-1} \dot{x}_e \quad (15)$$

and its time derivative

$$\ddot{q} = J_e^{-1} \ddot{x}_e + \frac{d}{dt} (J_e^{-1}) \dot{x}_e. \quad (16)$$

The merit of the description in an *extended task space* is, that the differential kinematics of redundant manipulators can be treated as if they were non-redundant manipulators.

Given an end-effector trajectory  $u_d$ , which we wish to track, and a null-space trajectory  $\dot{x}_{nd}$ , which can be used for additional subtasks, the *forward kinematics* and *inverse kinematics* is immediately available

$$\dot{x}_{ed} = \begin{bmatrix} u_d \\ \dot{x}_{nd} \end{bmatrix} = J_e \dot{q}_d, \quad \dot{q}_d = J_e^{-1} \begin{bmatrix} u_d \\ \dot{x}_{nd} \end{bmatrix}, \quad (17)$$

where  $u_d$ ,  $\dot{x}_{nd}$ , and  $\dot{q}_d$  are the desired end-effector velocity, the null-space velocity, and the corresponding joint velocity, respectively. There are a few methods available to obtain the desired null-space velocity  $\dot{x}_{nd}$ . Depending on the application it may be defined as

$$\dot{x}_{nd} = K_{2nd} J_n h, \quad h = \nabla H, \quad (18)$$

where  $K_{2nd}$  is a constant,  $J_n$  is the null-space Jacobian, and  $h$  and  $H$  specifies the secondary performance [6].

#### B. Dynamic Visual Feedback System for redundant manipulators

We are given a description of the dynamics of a robot manipulator in the form of the equation

$$M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + N(q) = \tau + \tau_d, \quad (19)$$

where  $q$ ,  $\dot{q}$  and  $\ddot{q}$  are the joint angles, velocities, and accelerations, respectively.  $\tau \in \mathbb{R}^n$  is the vector of the input torques,  $\tau_d \in \mathbb{R}^n$  is a disturbance input,  $M(q) \in \mathbb{R}^{n \times n}$  is the joint inertia matrix,  $C(q, \dot{q}) \dot{q} \in \mathbb{R}^n$  are the Coriolis and centrifugal torques,  $N(q) \in \mathbb{R}^n$  is the gravity.

Now, let us define a *null-space force* vector  $f_n$  responsible for the null-space motion  $\dot{x}_n$  and a *task-space force*  $f_b$  responsible for the end-effector motion. Then, by use of the principle of virtual work we obtain the following relation:

$$\tau = J_e^T F_e = J_b^T f_b + J_n^T f_n, \quad F_e = \begin{bmatrix} f_b \\ f_n \end{bmatrix}, \quad (20)$$

where  $F_e$  consists of the forces in *extended task space* [7].

When we consider the motion control of a redundant manipulator, it is more convenient to express the dynamic

equations in the task space form. This form can be easily derived using (15), (16), (19), and (20) as

$$M_e(q) \ddot{x}_e + C_e(q, \dot{q}) \dot{x}_e + N_e(q) = F_e + J_e^{-T} \tau_d, \quad (21)$$

where

$$\begin{aligned} M_e &= J_e^{-T} M J_e^{-1}, & N_e &= J_e^{-T} N \\ C_e &= J_e^{-T} \left( C J_e^{-1} + M \frac{d}{dt} (J_e^{-1}) \right). \end{aligned} \quad (22)$$

It can be shown that if  $\dot{M} - 2C$  is a skew-symmetric matrix then the matrix  $\dot{M}_e - 2C_e$  is also skew-symmetric, i.e.,  $z^T (\dot{M}_e - 2C_e) z = 0$ , where  $z$  is an arbitrary vector.

Turning to [7] reveals that there exists an inertial coupling effect between the task space and null space, which means that the null-space motion induces an additional task-space motion term, and vice versa. Using  $(J_n M J_n^T)^{-1} J_n M$  instead of  $J_n$ , the dynamic of the task-space motion can be decoupled from the null-space dynamic.

Let us define the error vector with respect to the extended task space of the manipulator dynamics as  $\xi_e := \dot{x}_e - \dot{x}_{ed}$ . Here, we define the weight matrices  $W_c := \text{diag}\{\omega_{pc} I_3, \omega_{rc} I_3\} \in \mathbb{R}^{6 \times 6}$  and  $W_e := \text{diag}\{\omega_{pe} I_3, \omega_{re} I_3\} \in \mathbb{R}^{6 \times 6}$  where  $\omega_{pc}, \omega_{rc}, \omega_{pe}, \omega_{re} \in \mathbb{R}$  are positive. Now, we consider the passivity-based dynamic visual feedback control law as follows

$$\begin{aligned} F_e &= M_e(q) \ddot{x}_{ed} + C_e(q, \dot{q}) \dot{x}_{ed} + N_e(q) \\ &+ J_e^{-T} J_b^T \text{Ad}_{(g_d^{-1})}^T W_c e_c + u_{\xi_e}. \end{aligned} \quad (23)$$

The new input  $u_{\xi_e}$  is to be determined in order to achieve the control objectives.

Using (10), (21) and (23), the visual feedback system with manipulator dynamics (we call the dynamic visual feedback system for redundant manipulators) can be derived as follows

$$\begin{aligned} \begin{bmatrix} \dot{\xi}_e \\ V_{ec}^b \\ V_{ee}^b \end{bmatrix} &= \begin{bmatrix} -M_e^{-1} C_e \xi_e + M_e^{-1} J_e^{-T} J_b^T \text{Ad}_{(g_d^{-1})}^T W_c e_c \\ -\text{Ad}_{(\bar{g}_{co}^{-1})} J_b J_e^{-1} \xi_e \\ 0 \end{bmatrix} + \\ &\begin{bmatrix} M_e^{-1} & 0 & 0 \\ 0 & -\text{Ad}_{(\bar{g}_{co}^{-1})} & I \\ 0 & 0 & -\text{Ad}_{(g_{ee}^{-1})} \end{bmatrix} u + \begin{bmatrix} M_e^{-1} J_e^{-T} & 0 \\ 0 & 0 \\ 0 & I \end{bmatrix} w, \end{aligned} \quad (24)$$

where  $\chi := [\xi_e^T \ e_c^T \ e_e^T]^T$  and  $u := [u_{\xi_e}^T \ u_d^T \ u_e^T]^T$ . We define the disturbance of dynamic visual feedback system as  $w := [\tau_d^T \ (V_{wo}^b)^T]^T$ .

It should be noted that if the vectors of control error  $e_c$ , estimation error  $e_e$  and error  $\xi_e$  are equal to zero, then the estimated relative rigid body motion  $\bar{g}_{co}$  equals the reference  $g_d$ , the estimated relative rigid body motion  $\bar{g}_{co}$  equals the actual one  $g_{co}$ , and the extended end-effector velocity  $\dot{x}_e$  equals the desired one  $\dot{x}_{ed}$  (including the desired null-space velocity  $\dot{x}_{nd}$ ). The block diagram of the dynamic of the redundant manipulator and the proposed control law is shown in Fig. 2

Before constructing the dynamic visual feedback control law, we derive an important lemma.

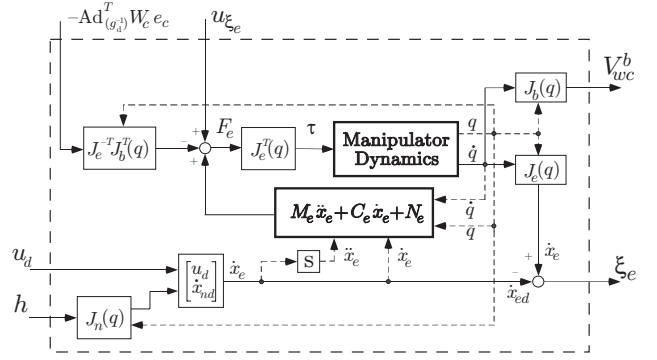


Fig. 2. Block diagram of the manipulator dynamics and the controller

*Lemma 1:* if  $w = 0$ , then the dynamic visual feedback system (24) satisfies

$$\int_0^T u^T \nu d\tau \geq -\beta, \quad \forall T > 0, \quad (25)$$

where

$$\nu := N \chi, \quad N := \begin{bmatrix} I & 0 & 0 \\ 0 & -\text{Ad}_{(g_d^{-1})}^T W_c & 0 \\ 0 & \text{Ad}_{(e^{-\xi \theta_{ec}})} W_c & -W_e \end{bmatrix}. \quad (26)$$

*Proof:* Consider the following definite function

$$\begin{aligned} V &= \frac{1}{2} \xi_e^T M_e \xi_e + \frac{1}{2} w_{pc} \|p_{ec}\|^2 + w_{rc} \phi(e^{\xi \theta_{ec}}) \\ &+ \frac{1}{2} w_{pe} \|p_{ee}\|^2 + w_{re} \phi(e^{\xi \theta_{ee}}). \end{aligned} \quad (27)$$

where  $\phi(e^{\xi \theta}) := \frac{1}{2} \text{tr}(I - e^{\xi \theta})$  is the error function of the rotation matrix (see, e.g., [4][10]). Differentiating (27) with respect to time yields

$$\begin{aligned} \dot{V} &= \frac{1}{2} \xi_e^T \dot{M}_e \xi_e \\ &+ \chi^T \begin{bmatrix} M_e(q) & 0 & 0 \\ 0 & W_c \text{Ad}_{(e^{\xi \theta_{ec}})} & 0 \\ 0 & 0 & W_e \text{Ad}_{(e^{\xi \theta_{ee}})} \end{bmatrix} \begin{bmatrix} \dot{\xi}_e \\ V_{ec}^b \\ V_{ee}^b \end{bmatrix}. \end{aligned} \quad (28)$$

The above equation along the trajectories of the system (24) can be transformed into

$$\dot{V} = \chi^T \begin{bmatrix} I & 0 & 0 \\ 0 & -W_c \text{Ad}_{(g_d^{-1})} & W_c \text{Ad}_{(e^{\xi \theta_{ec}})} \\ 0 & 0 & -W_e \end{bmatrix} u. \quad (29)$$

Integrating (29) from 0 to  $T$ , we can obtain

$$\int_0^T u^T \nu d\tau = V(T) - V(0) \geq -V(0) := -\beta, \quad (30)$$

where  $\beta$  is the positive scalar which only depends on the initial states of  $\xi_e$ ,  $g_{ec}$  and  $g_{ee}$ .

*Remark 1:* The visual feedback system (24) satisfies the passivity property. It is well known that the manipulator dynamics (19) also has the passivity. These passivity properties are connected by the manipulator Jacobian (12) or (15). In

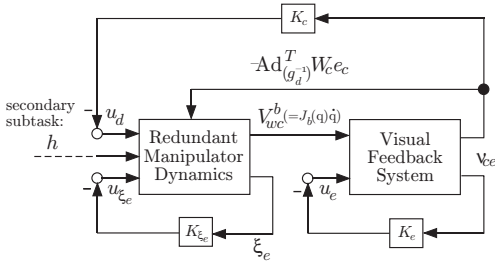


Fig. 3. Block diagram of the dynamic visual feedback control

Lemma 1, the inequality (25) would suggest that the dynamic visual feedback system (24) is *passive* from the input  $u$  to the output  $\nu$ .

### C. Stability Analysis

It is well known that there is a direct link between passivity and Lyapunov stability. Thus, we propose the following control input

$$u = -K\nu = -KN\chi, \quad K := \begin{bmatrix} K_{\xi_e} & 0 & 0 \\ 0 & K_c & 0 \\ 0 & 0 & K_e \end{bmatrix}, \quad (31)$$

where  $K_{\xi_e}$  is a  $n \times n$  positive definite matrix,  $K_c$  and  $K_e$  are  $6 \times 6$  positive definite matrices. The result with respect to asymptotic stability of the proposed control input (31) can be established as follows.

*Theorem 1:* If  $w = 0$ , then the equilibrium point  $\chi = 0$  for the closed-loop system (24) and (31) is asymptotic stable.

*Proof:* In the proof of Lemma 1, we have already derived that the time derivative of  $V$  along the trajectory of the system (24) is formulated as (29). Using the control input (31), (29) can be transformed into

$$\dot{V} = -\chi^T N^T K N \chi, \quad (32)$$

This completes the proof.

The block diagram of the dynamic visual feedback control is shown in Fig. 3.

## IV. SIMULATION

In this section, the simulation results on the redundant manipulator as depicted in Fig. 1 are shown in order to understand our proposed method. Though our proposed method is valid for 3D visual feedback systems, for simplicity, we will consider a 4-DOF planar redundant manipulator. The end-effector is able to track the moving target object with respect to both the position in  $xy$ -plane and the orientation  $\psi$ , see Fig. 1. Hence the manipulator can be considered as redundant, i.e. the degree of redundancy is  $r = 1$ .

The target object has four feature points and moves along a straight line ( $0 \leq t \leq 5$  [s]). Specifically, we use the reference of the relative rigid body motion as a constant value, i.e.  $p_d = [0 \ 0 \ -0.81]^T$ , and  $e^{\xi\theta_d} = I$ . Furthermore, the target object rotates with an average angular velocity  $\bar{\omega}_z = 0.5$  [rad/s]. The link lengths are  $L_1 = L_2 = 1.0$  [m] and  $L_3 = 0.3$  [m], the link masses are  $m_1 = m_2 = 9.5$  [kg] and  $m_3 = 2$  [kg], and the link inertias are  $I_{z1} = I_{z2} =$

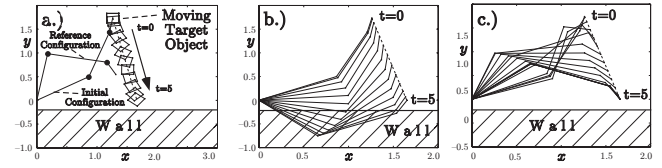


Fig. 4. Obstacle avoidance (a) simulation framework, arm configuration for  $K_{2nd} = 0$  (b) and  $K_{2nd} = 2$  (c)

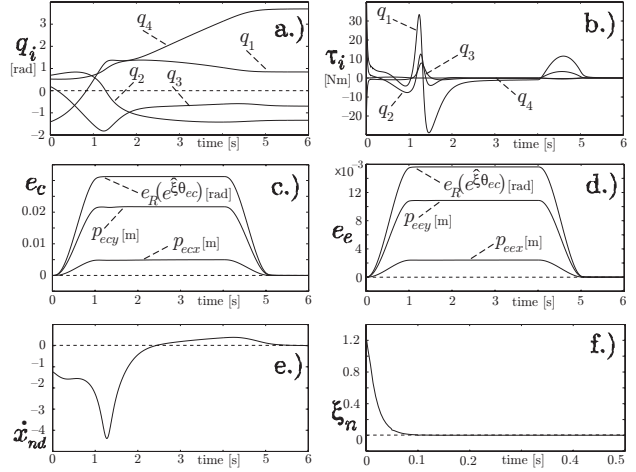


Fig. 5. Obstacle avoidance (a) joint position, (b) joint torque, (c) control error  $e_c$ , (d) estimation error  $e_e$ , (e) desired null-space velocity  $\dot{x}_{nd}$ , and (f) null-motion error  $\xi_n$

$1.147$  [kg · m<sup>2</sup>], and  $I_{z3} = 0.5182$  [kg · m<sup>2</sup>]. The mass of the camera is  $m_4 = 2$  [kg] and the mass moment of inertia is  $I_{z4} = 0.5123$  [kg · m<sup>2</sup>].

The control gains of the redundant manipulator are chosen as  $K_{\xi_e} = 20I_4$ ,  $K_e = 50I_6$ , and  $K_c = \text{diag}\{2, 2, 1, 1, 1, 2\} \times 10^2$ . Also, we select the weight matrices for control design as  $W_c = 0.1I_6$  and  $W_e = I_6$ .

**Simulation I:** First, the obstacle avoidance control is simulated with the desired trajectory shown in Fig. 4(a). The task is divided into a first subtask, *tracking the moving target object*, and a second subtask, *avoiding the obstacle* (wall). Though we can apply various complex obstacle avoidance strategies, for simplicity, the specification of the null-space velocity is given by  $\nabla H = K_{2nd}(q_{ref} - q)$ , where  $K_{2nd}$  is a constant and  $q_{ref}$  is a reference configuration taught by a human operator beforehand in order to avoid a particular obstacle. The initial configuration is  $q_0 = [0.35 \ 0.52 \ 0.35 \ 0]^T$  [rad] and the reference configuration for avoiding the obstacle is given by  $q_{ref} = [1.39 \ -1.57 \ -0.87 \ 0]^T$  [rad]; illustrated in Fig. 4(a).

From the simulation results in Fig. 4(c) it is apparent that the end-effector tracks the moving target object while avoiding the obstacle. Whereas, in Fig. 4(b), without exploiting redundancy the robot manipulator would collide with the obstacle. Fig. 5 presents the joint positions, joint torques, the error vectors  $e_c$  and  $e_e$  of the relative rigid body motion, the desired null-space velocity  $\dot{x}_{nd}$  and the null-motion error  $\xi_n = \dot{x}_n - \dot{x}_{nd}$ , respectively.



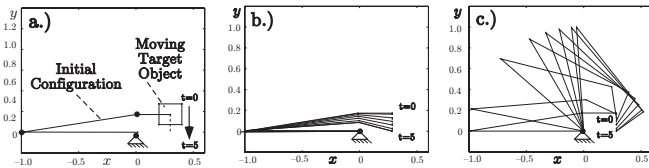


Fig. 6. Singularity avoidance (a) simulation framework, arm configuration for  $K_{2nd} = 0$  (b) and  $K_{2nd} = 50$  (c)

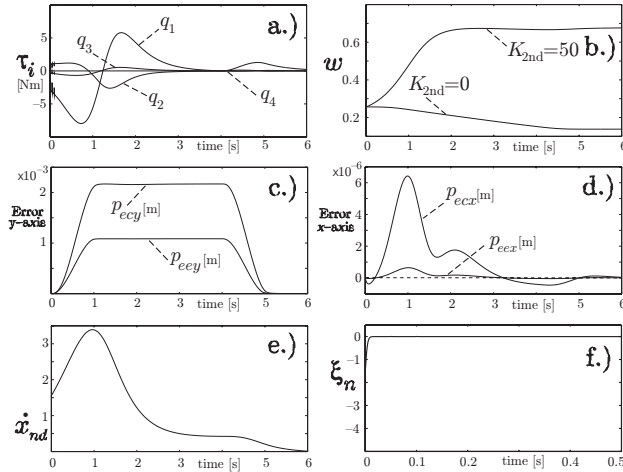


Fig. 7. Singularity avoidance (a) joint torque, (b) manipulability measure for  $K_s = 0$  and  $K_s = 50$ , control and estimation error (c) y-axis and (d) x-axis, (e) desired null-space velocity  $\dot{x}_{nd}$ , and (f) null-motion error  $\xi_n$

**Simulation II:** Second, the singularity avoidance control is simulated with the desired trajectory shown in Fig.6(a). The initial configuration is given by  $q_0 = [3.14 \ -2.97 \ -0.17 \ 0]^T$  [rad]. As a secondary performance  $H$ , we use the manipulability measure  $w = \sqrt{\det(JJ^T)}$ . Hence, the task is divided into a first subtask, *tracking the moving target object*, and a second subtask, *avoiding singularities* by maximizing  $w$ .

Fig. 6(b) shows the simulation result for the object tracking problem without exploiting redundancy. Whereas in Fig.6(c) the end-effector tracks the moving target object while exploiting redundancy by maximizing the manipulability measure  $w$ . Fig.7 presents the joint torques, manipulability measure for  $K_{2nd} = 0$  (no singularity avoidance) and  $K_{2nd} = 50$  (singularity avoidance), the error vectors  $e_c$  and  $e_e$  of the relative rigid body motion, the desired null-space motion  $\dot{x}_{nd}$  and the null-motion error  $\xi_n = \dot{x}_n - \dot{x}_{nd}$ , respectively.

**Stability of end-effector and null-space motion:** In the case of the static target object, i.e.  $t = 5s$ , for both the obstacle and singularity avoidance control, all errors in Fig.5(c)(d) and Fig.7(c)(d) tend to zero. Thus, it can be concluded that if the target object is static the end-effector tracks the moving target object and moreover the equilibrium point is asymptotically stable.

Furthermore, in Fig. 5 (e)(f) and Fig. 7 (e)(f) one can easily observe that the desired null-motion  $\dot{x}_{nd}$  and the null-motion

error  $\xi_n$  decreases exponentially to zero, yielding good null-motion capability without affecting the end-effector motion.

After all, the simulation results show that our proposed control law for dynamic visual feedback systems can be extended for manipulators with kinematic redundancy.

## V. CONCLUSIONS

In this paper the dynamic visual feedback control of redundant manipulators for the 3D target tracking has been investigated. We have considered an extended task space control law for kinematically redundant manipulators which ensures object tracking as well as the control of null-motion by maximizing a performance index. For this reason, our proposed passivity approach for redundant manipulator will enrich not only the field of visual feedback control but also the field of robot control for redundant manipulators.

The effectiveness of the proposed control method was verified by means of various simulations for possible applications such as obstacle avoidance or singularity avoidance. Future work will be devoted to show the performance by means of experimental results. One possible main challenge we need to cope with is the computational complexity of the proposed control algorithm.

## VI. ACKNOWLEDGMENTS

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