Infinite-Horizon Sequence-based Networked Control without Acknowledgments

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***Abstract*— In this paper, we consider infinite-horizon networked LQG control over multipurpose networks that do not provide acknowledgments (UDP-like networks). The information communicated over the network experiences transmission delays and losses that are modeled as stochastic processes. In oder to mitigate the delays and losses in the controller-actuator channel, the controller transmits sequences of predicted control inputs in addition to the current control input. To be able to reduce the impact of delays and losses in the feedback channel, the estimator computes the estimate using the M last measurements. In this scenario, the separation principle does not hold and the optimal control law is in general nonlinear. However, we show that by restricting the controller and the estimator to linear systems with constant gains, we can find the optimal solution. The presented control law is demonstrated in a numerical example.**

I. INTRODUCTION

In Network Control Systems (NCS), connections between system components are provided via networks. Although there exist networks that were specifically designed for control purposes and thus, can guarantee reliable transmission characteristics, there is a desire in using multipurpose networks such as WLAN or the Internet. This interest is motivated by the high availability and the non-proprietary nature of the multipurpose networks, and the ability to even perform wireless control. However, multipurpose networks introduce negative effects into the control loop such as limited bandwidth, packet delay and loss, quantization, etc. These effects can degrade system performance or even lead to instability if not anticipated [1], [2], [3]. Therefore, a large variety of control methods was developed to address these issues. In the following, we restrict ourselves to packet delays and losses and assume that these effects can be modeled by stochastic processes. We refer to the networks with this delay and loss model as *stochastic networks*.

If the networks are assumed to be capable of transmitting more information than only a single control input per data packet, *sequence-based* control can be applied to cope with packet delays and losses. The main idea of this control method, introduced in [4], is to let the controller transmit a sequence of predicted control inputs along with the current control input. By doing so, the actuator can use the additional information when subsequent packets are delayed or lost. Most approaches to sequence-based control can be associated with (i) the extension of a nominal controller, (ii) Model Predictive Control (MPC), or (iii) sequence-based Linear Quadratic Gaussian (LQG) control.

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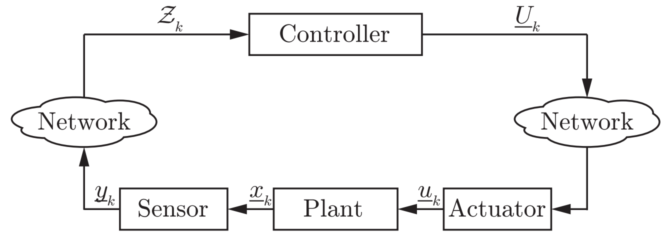

Fig. 1. Considered system setup. The linear plant is controlled over digital connections that are subject to information delay and loss.

The notion of (i), e.g., considered in [5], [6], consists of designing a nominal controller neglecting the networks and then adapting the controller in such a way that it considers the network-induced effects. Although this is a very convenient approach as existing controllers can be reused, the resulting controller is no longer optimal even if the nominal is optimal.

From the perspective of MPC (option (ii)), sequence-based control poses a natural extension as sequences of control inputs that minimize a finite-horizon control problem are already generated as a "byproduct". The main challenge here is to design the state prediction such that it is consistent with the actually possible state evolution [7], [8], [9]. Sequence-based control originating from MPC has the advantage of being able to take into account state and input constraints. However, it requires an optimization problem to be solved at each time step.

Option (iii) to address sequence-based control is to state an LQG control problem and to minimize the cost function directly w.r.t. complete sequences. This approach was considered in [10] with packet losses only and in our previous publication [11] with both packet delays and losses. In [11], we derived an optimal linear controller under the assumption that the controller receives instantaneous acknowledgment of successfully transmitted sequences. For this purpose, we modeled the NCS as a Markov Jump Linear System (MJLS) and applied *Dynamic Programming* in order to minimize the cost function.

In this paper, we consider a setup similar to [11]. However, we do not pose the requirement of acknowledgments on successful transmissions in contrast to [10], [11]. Moreover, the controller does not receive any kind of transmission acknowledgments. In this case, separation between control and estimation does not hold [12] and thus, approximations have to be conducted. Following the argumentation in [12], we make the assumption that the estimator has constant gains and finite memory, and the controller is linear with constant gains.

Outline: The remainder of this paper is organized as follows. In Sec. II, we formulate the considered problem. The challenges of the considered problem and the proposed solution are discussed in Sec. III. The control law that solves the problem, is derived in Sec. IV and evaluated in Sec. V. Finally, Sec. VI concludes the paper.

Notation: Throughout this paper, vectors are underlined $\underline{x}$, random variables are in bold letters $\boldsymbol{a}, \underline{\boldsymbol{x}}$, and matrices are in bold capital letters $\mathbf{M}$. A transpose of a matrix $\mathbf{A}$ is denoted by $\mathbf{A}^\top$ and its Moore-Penrose pseudoinverse by $\mathbf{A}^\dagger$. To indicate positive definiteness and positive semidefiniteness of a matrix $\mathbf{A}$, we write $\mathbf{A} > 0$ and $\mathbf{A} \geq 0$, respectively. A matrix with all elements being zero is denoted by $\mathbf{0}$ and the identity matrix by $\mathbf{I}$. A sequence $\{a_0, a_1, \dots, a_n\}$ is abbreviated by $a_{0:n}$.

II. PROBLEM FORMULATION

In this paper, we address the problem of controlling a linear plant over a digital network. The structure of the control loop with time-triggered and synchronized components is depicted in Fig. 1. The discrete-time dynamics of the plant are given by

$$\underline{\boldsymbol{x}}_{k+1} = \mathbf{A}\underline{\boldsymbol{x}}_k + \mathbf{B}\underline{\boldsymbol{u}}_k + \underline{\boldsymbol{w}}_k \ , \tag{1}$$

where the state is denoted by $\underline{\boldsymbol{x}}_k \in \mathbb{R}^n$, the control input by $\underline{\boldsymbol{u}}_k \in \mathbb{R}^m$, and the process noise by $\underline{\boldsymbol{w}}_k$. The state $\underline{\boldsymbol{x}}_k$ is measured according to

$$\underline{\boldsymbol{y}}_k = \mathbf{C}\underline{\boldsymbol{x}}_k + \underline{\boldsymbol{v}}_k \ , \tag{2}$$

where $\underline{\boldsymbol{y}}_k \in \mathbb{R}^p$ denotes the measurement at time step k and $\underline{\boldsymbol{v}}_k$ the measurement noise. Both noises $\underline{\boldsymbol{w}}_k$ and $\underline{\boldsymbol{v}}_k$ are assumed to be i.i.d. zero-mean Gaussian with covariances

$$\mathbf{W} = \mathrm{E}\left\{\underline{\boldsymbol{w}}_k \underline{\boldsymbol{w}}_k^\top\right\} \quad \text{and} \quad \mathbf{V} = \mathrm{E}\left\{\underline{\boldsymbol{v}}_k \underline{\boldsymbol{v}}_k^\top\right\} \ .$$

Furthermore, the initial state $\underline{\boldsymbol{x}}_0$ is also Gaussian with

$$\mathrm{E}\left\{\underline{\boldsymbol{x}}_0\right\} = \overline{\underline{x}}_0 \quad \text{and} \quad \mathrm{E}\left\{(\underline{\boldsymbol{x}}_0 - \overline{\underline{x}}_0)(\underline{\boldsymbol{x}}_0 - \overline{\underline{x}}_0)^\top\right\} = \mathbf{X}_0 \ ,$$

and the matrices $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ are known.

The control inputs are communicated to the actuator, which resides directly at the plant, over a controller-actuator network (CA-link) capable of transmitting time-stamped data packets $\underline{U}_k$. The packets $\underline{U}_k$ can experience (unbounded) time delays and losses. The network does not provide any sort of information on transmission success. Such networks are referred to as UDP-like[1] networks. The i.i.d. delays $\boldsymbol{\tau}_k^{CA} \in \mathbb{N}_0$ in the CA-link are modeled as a stationary stochastic process with a given probability density function (pdf) $f^{CA}(\boldsymbol{\tau}_k^{CA})$. We model packet losses as infinite delays. In order to compensate for control packet delays and losses, the controller transmits $N-1$ predicted control inputs $N \in \mathbb{N}_+$ in addition to the current control input at time step k. Thus, the control packet $\underline{U}_k$ with the length N is given by

$$\underline{U}_k = \begin{bmatrix} \underline{u}_{k|k}^\top & \underline{u}_{k+1|k}^\top & \cdots & \underline{u}_{k+N-1|k}^\top \end{bmatrix}^\top ,$$

where the notation $\underline{u}_{k+i|k}, i = 0, \dots, N-1$ denotes that the control input is generated at time step k and is intended to be applied to the plant at time step $k+i$. From this point, we will refer to control packets $\underline{U}_k$ as *control sequences*.

The actuator is equipped with a buffer, where it stores the most recent control sequence, i.e., the control sequence with the largest time index k among all received sequences. By doing so, the actuator can apply predicted control inputs from the buffered control sequence, if subsequent transmissions are delayed or lost. In case the buffer runs empty, the actuator applies the default control input $\underline{u}^d = \underline{0}$. Generalization to other default inputs is possible within the framework presented in Sec. IV but omitted for brevity.

The state $\underline{\boldsymbol{x}}_k$ is measured at each time step k and the measurements are transmitted to the controller over (probably) another UDP-like network (SC-link). In this network, the communicated packets can also experience (unbounded) i.i.d. delays and losses $\boldsymbol{\tau}_k^{SC} \in \mathbb{N}_0$ (losses correspond to infinite delays), which are modeled by a stationary stochastic process with a given pdf $f^{SC}(\boldsymbol{\tau}_k^{SC})$. Furthermore, the sensor only transmits the measurement $\underline{\boldsymbol{y}}_k$ at time step k and does not retransmit any previous measurements. The set of measurements received by the controller at time step k will be denoted by $\mathcal{Z}_k$. This set can contain one, none, or several measurements due to the stochastic nature of the SC-link.

The performance of the control is assessed by the infinite-horizon quadratic cost function

$$J_\infty = \lim_{K\to\infty} \frac{1}{K} \mathrm{E}\left\{ \sum_{k=0}^{K} \left(\underline{\boldsymbol{x}}_k^\top \mathbf{Q} \underline{\boldsymbol{x}}_k + \underline{u}_k^\top \mathbf{R} \underline{u}_k \right) \middle| \mathcal{I}_0 \right\} \ , \tag{3}$$

where the design parameters $\mathbf{Q} \geq 0$ and $\mathbf{R} > 0$ are of appropriate dimensions, $K \in \mathbb{N}_+$ is the optimization horizon, and $\mathcal{I}_k$ denotes the information set

$$\mathcal{I}_k = \left\{ \overline{\underline{x}}_0, \underline{U}_{0:k-1}, \mathcal{Z}_{1:k} \right\}$$

available to the controller.

We seek to find a control law that minimizes (3) w.r.t. the sequences $\underline{U}_k$. A solution to this problem is proposed in the next section.

III. PROPOSED SOLUTION

In this section, we discuss the two challenges of the control law derivation for the considered problem and illustrate the proposed solution. These two challenges are: (i) the feasibility of the optimal estimator and (ii) the *separation principle*.

According to the cost function (3), the control task is to stabilize the state $\underline{\boldsymbol{x}}_k$ around the origin. To be able to do this, the state information has to be recovered from the received measurements in form of a state estimate, which

[1] In NCS literature, the networks are denoted UDP-like if they do not provide acknowledgments. On the other hand, networks that provide instantaneous acknowledgments are denoted as TCP-like. Both terms do not refer to real UDP/IP and TCP/IP networks. However, it is important to point out that the assumption of TCP-like networks is not suitable for real-world applications already because the acknowledgments may experience loss and delay.

then can be used to compute control sequences $\underline{U}_k$. However, as we make no assumption that the packet delays in the SC-channel are bounded, the optimal estimation law is infeasible because it would require infinite memory [13]. This issue poses challenge (i). A possible solution to this problem is to implement a time-invariant estimator with finite memory. Thus, we make Assumption 1.

Assumption 1 *The controller uses constant gains to compute the state estimate based on available measurements. Furthermore, it only accepts and uses measurements whose delay satisfies*

$$\boldsymbol{\tau}_k^{SC} \leq M \ , \quad M \in \mathbb{N} \ .$$

Measurements with longer delay are discarded.

Assumption 1 can be guaranteed if the sensor transmits time-stamped measurements to the controller. Then, the measurement delay can be computed as the difference between the current time and the measurement time stamp because the control loop components are synchronized.

Challenge (ii) was considered in [14], where the authors address the considered problem of minimizing (3) if only losses are present. They demonstrate that due to the UDP-like property of the CA-channel, the separation principle does not hold, i.e., the estimation error is not independent of the control inputs, except for some very restrictive system classes. The optimal control law is thus nonlinear. However, the derivation of an optimal linear controller with constant gains is possible. For this purpose, we require the following assumption to hold.

Assumption 2 *The control law is linear in the state estimate and has constant gains.*

With these assumptions, we derive the optimal linear sequence-based control law in the following section.

IV. CONTROL LAW DERIVATION

In this section, we derive the optimal linear control law. For this purpose, we first formulate the considered NCS as a Markov Jump Linear System (MJLS) using state augmentation. Then, in Sec. IV-B, we model the closed-loop dynamics based on Assumptions 1 and 2. The actual control law derivation is then given in Sec. IV-C.

A. Augmented System Dynamics

In order to formulate the considered NCS as a MJLS, we proceed as follows. First, we introduce the vector $\underline{\eta}_k$ that contains control inputs, which can be applied to the plant at future time steps, and model the situation at the actuator as a stochastic dynamical system. Then, to be able to model the situation at the controller, i.e., the measurement arrival and the state estimation, we introduce the vector $\underline{\psi}_k$ that contains previous states and the vector $\underline{\boldsymbol{Y}}_k$ that contains relevant measurements. Finally, we combine the individual models to obtain the process and measurement dynamics of the complete NCS.

As indicated above, we introduce the vector $\underline{\eta}_k$, which includes all control inputs from the sequences $\underline{U}_{k-N+1:k-1}$ that still can be applied to the plant. A more detailed version of this modeling approach is available in [11]. Thus, we only summarize the formulation of $\underline{\eta}_k$. Formally, $\underline{\eta}_k$ is given by

$$\underline{\eta}_k = \begin{bmatrix} \begin{bmatrix} \underline{u}_{k|k-1}^\top & \underline{u}_{k+1|k-1}^\top & \cdots & \underline{u}_{k+N-2|k-1}^\top \end{bmatrix}^\top \\ \begin{bmatrix} \underline{u}_{k|k-2}^\top & \underline{u}_{k+1|k-2}^\top & \cdots & \underline{u}_{k+N-3|k-2}^\top \end{bmatrix}^\top \\ \vdots \\ \underline{u}_{k|k-N+1} \end{bmatrix}^\top .$$

For the dynamics of $\underline{\eta}_k$, it holds

$$\underline{\eta}_{k+1} = \mathbf{F}\underline{\eta}_k + \mathbf{G}\underline{U}_k \ ,$$

with

$$\mathbf{F} = \begin{array}{c} \begin{array}{ccccccc} \#columns{:}m & m(N-1) & m & m(N-2) & & m & m \end{array} \\ \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I} & \mathbf{0} \end{bmatrix} \end{array} \begin{array}{l} \#rows{:} \\ \}mN \\ \}m(N-1) \\ \}m(N-2) \\ \\ \}m \end{array} ,$$

$$\mathbf{G} = \begin{array}{c} \begin{array}{cc} \#columns{:}m & m(N-1) \end{array} \\ \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \end{array} \begin{array}{l} \#rows{:} \\ \}m(N-1) \\ \}m\frac{(N-1)(N-2)}{2} \end{array} .$$

At this point, we introduce the random variable $\boldsymbol{\theta}_k \in \{0, 1, \ldots, N\}$ that describes the age of the currently buffered control sequence at the actuator. Suppose, that the most recent sequence, i.e., the sequence with the largest time index, received by the actuator is $\underline{U}_t, t \in \mathbb{N}_0, t \leq k$. Then, for $\boldsymbol{\theta}_k$ it holds

$$\boldsymbol{\theta}_k = \min\left(k - t, N\right) \ ,$$

where $\boldsymbol{\theta}_k = N$ corresponds to the case that there are no buffered control inputs that can be applied to the plant and thus the default input will be applied. With this definition of $\boldsymbol{\theta}_k$, the control input that is actually applied to the plant is selected according to

$$\underline{\boldsymbol{u}}_k = \mathbf{H}_k \underline{\eta}_k + \mathbf{J}_k \underline{U}_k \ ,$$

with

$$\mathbf{H}_k = \begin{array}{c} \begin{array}{cccccc} \#columns{:}\ m & m(N-2) & m & m(N-3) & & m \end{array} \\ \begin{bmatrix} \delta_{\boldsymbol{\theta}_k,1}\mathbf{I} & \mathbf{0} & \delta_{\boldsymbol{\theta}_k,2}\mathbf{I} & \mathbf{0} & \ldots & \delta_{\boldsymbol{\theta}_k,N-1}\mathbf{I} \end{bmatrix} \end{array} \begin{array}{l} \#rows{:} \\ \}m \end{array} ,$$

$$\mathbf{J}_k = \begin{array}{c} \begin{array}{cc} \#columns{:}\ m & m(N-1) \end{array} \\ \begin{bmatrix} \delta_{\boldsymbol{\theta}_k,0}\mathbf{I} & \mathbf{0} \end{bmatrix} \end{array} \begin{array}{l} \#rows{:} \\ \}m \end{array} ,$$

where $\delta_{\boldsymbol{\theta}_k,i} = 1$ if $\boldsymbol{\theta}_k = i$ and 0 otherwise. Note that the matrices $\mathbf{H}_k$ and $\mathbf{J}_k$ are stochastic and thus the actual control input $\underline{\boldsymbol{u}}_k$ is a random variable.

Having obtained the model of the situation at the actuator, we now model the situation at the estimator. For this purpose, we introduce the vector $\underline{\psi}_k$ that contains relevant previous states $\underline{\boldsymbol{x}}_{k-M+1:k-1}$

$$\underline{\psi}_k = \begin{bmatrix}\underline{\boldsymbol{x}}_{k-1}^\top & \underline{\boldsymbol{x}}_{k-2}^\top & \cdots & \underline{\boldsymbol{x}}_{k-M+1}^\top\end{bmatrix}^\top .$$

The dynamics of $\underline{\psi}_k$ are given by

$$\underline{\psi}_{k+1} = \mathbf{D}\underline{\boldsymbol{x}}_k + \mathbf{E}\underline{\psi}_k ,$$

with

$$\mathbf{D} = \begin{bmatrix}\mathbf{I}\\ \mathbf{0}\\ \mathbf{0}\\ \vdots\\ \mathbf{0}\end{bmatrix} , \quad \mathbf{E} = \begin{bmatrix}\mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0}\\ \mathbf{I} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I} & \dots & \mathbf{0} & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{I} & \mathbf{0}\end{bmatrix} ,$$

where the identity and zero matrices have the dimension $n \times n$.

The measurements considered when computing the state estimate at time step k are included in the vector $\underline{\boldsymbol{Y}}_k$ defined according to

$$\underline{\boldsymbol{Y}}_k = \left[\underline{\boldsymbol{y}}_k{}^\top \quad \underline{\boldsymbol{y}}_{k-1}{}^\top \quad \cdots \quad \underline{\boldsymbol{y}}_{k-M+1}{}^\top\right]^\top .$$

Finally, introducing the augmented state vector $\underline{\boldsymbol{\xi}}_k$ with

$$\underline{\boldsymbol{\xi}}_k = \left[\underline{\boldsymbol{x}}_k^\top \quad \underline{\psi}_k^\top \quad \underline{\eta}_k^\top\right]^\top ,$$

we obtain the augmented process dynamics

$$\underline{\boldsymbol{\xi}}_{k+1} = \widetilde{\mathbf{A}}_k\underline{\boldsymbol{\xi}}_k + \widetilde{\mathbf{B}}_k\underline{U}_k + \underline{\widetilde{\boldsymbol{w}}}_k ,$$

with

$$\widetilde{\mathbf{A}}_k = \begin{bmatrix}\mathbf{A} & \mathbf{0} & \mathbf{BH}_k\\ \mathbf{D} & \mathbf{E} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{F}\end{bmatrix} , \ \widetilde{\mathbf{B}}_k = \begin{bmatrix}\mathbf{BJ}_k\\ \mathbf{0}\\ \mathbf{G}\end{bmatrix} , \ \underline{\widetilde{\boldsymbol{w}}}_k = \begin{bmatrix}\underline{\boldsymbol{w}}_k\\ \underline{0}\\ \underline{0}\end{bmatrix} .$$

And for the augmented measurement dynamics, it holds

$$\underline{\boldsymbol{Y}}_k = \widetilde{\mathbf{C}}_k\underline{\boldsymbol{\xi}}_k + \underline{\widetilde{\boldsymbol{v}}}_k , \tag{4}$$

with

$$\widetilde{\mathbf{C}}_k = \begin{bmatrix}\gamma_{k|k}\mathbf{C} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \gamma_{k|k-1}\mathbf{C} & \dots & \mathbf{0} & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ \mathbf{0} & \mathbf{0} & \dots & \gamma_{k|k-M+1}\mathbf{C} & \mathbf{0}\end{bmatrix} ,$$

$$\underline{\boldsymbol{v}}_k = \begin{bmatrix}\underline{\boldsymbol{v}}_k^\top & \underline{\boldsymbol{v}}_{k-1}^\top & \dots & \underline{\boldsymbol{v}}_k^\top\end{bmatrix}^\top ,$$

where the zero matrices are of dimension $p \times n$ and $\gamma_{k|k-i} = 1, i \in \{0, 1, \dots, M-1\}$ if the measurement $\underline{\boldsymbol{y}}_{k-i}$ is available to the controller and 0 otherwise. Please note that if $\gamma_{k|k-i} = 0$, the absent measurement is modeled as noise $\underline{\boldsymbol{v}}_{k-i}$. This is equivalent to completely removing $\underline{\boldsymbol{y}}_{k-i}$ from the vector $\underline{\boldsymbol{Y}}_k$ [15]. However, the method employed in this paper simplifies the calculations. The derived MJLS possesses the jumping parameters $\boldsymbol{\theta}_k$ and $\gamma_{k|k-i}$.

Remark 1 *Although the considered NCS is modeled as a MJLS, the MJLS approaches presented, e.g., in [16], [17], and [18], differ from our approach. The approaches [16], [17] do not consider stochastic system dynamics and non-observable mode or assume separation a priori, and the approach [18] is based on the direct state observation without any measurement noise.*

B. Closed-Loop System Dynamics

In this section, we derive the closed-loop dynamics of the considered NCS.

According to Assumption 1, the estimator possesses constant gains. Thus, the estimate of the augmented system state can be calculated according to

$$\underline{\widehat{\boldsymbol{\xi}}}_{k+1} = \widehat{\mathbf{A}}\underline{\widehat{\boldsymbol{\xi}}}_k + \widehat{\mathbf{B}}\underline{U}_k + \mathbf{K}\left(\underline{\boldsymbol{Y}}_k - \widetilde{\mathbf{C}}_k\underline{\widehat{\boldsymbol{\xi}}}_k\right) , \tag{5}$$

where $\underline{\widehat{\boldsymbol{\xi}}}_{k+1}$ is clearly based on the information available to the estimator at time step k, i.e., $\underline{\widehat{\boldsymbol{\xi}}}_{k+1} = \mathrm{E}\left\{\underline{\boldsymbol{\xi}}_{k+1}\middle|\mathcal{I}_k\right\}$. The constant estimation gain in (5) is denoted by $\mathbf{K}$, and for the matrices $\widehat{\mathbf{A}}$ and $\widehat{\mathbf{B}}$, it holds

$$\widehat{\mathbf{A}} = \mathrm{E}\left\{\widetilde{\mathbf{A}}_k\middle|\mathcal{I}_k\right\} \text{ and } \widehat{\mathbf{B}} = \mathrm{E}\left\{\widetilde{\mathbf{B}}_k\middle|\mathcal{I}_k\right\} .$$

The calculation of the expected values of $\widetilde{\mathbf{A}}_k$ and $\widetilde{\mathbf{B}}_k$ requires the steady-state probability distribution of $\boldsymbol{\theta}_k$. This limit distribution $\underline{\theta}$ exists, because the Markov chain whose state $\boldsymbol{\theta}_k$ represents is ergodic [19].

Following the Assumption 2, the control law is given by

$$\underline{U}_k = -\mathbf{L}\underline{\widehat{\boldsymbol{\xi}}}_k ,$$

where $\mathbf{L}$ is the constant control gain.

With this prerequisites, and the definition

$$\underline{\boldsymbol{\chi}}_k = \left[\underline{\boldsymbol{\xi}}_k^\top \quad \underline{\widehat{\boldsymbol{\xi}}}_k^\top\right]^\top ,$$

the closed-loop dynamics of the NCS can be written in terms of the augmented state as

$$\underline{\boldsymbol{\chi}}_{k+1} = \mathcal{A}_k\underline{\boldsymbol{\chi}}_k + \underline{\boldsymbol{\mathcal{W}}}_k , \tag{6}$$

with

$$\mathcal{A}_k = \begin{bmatrix}\widetilde{\mathbf{A}}_k & -\widetilde{\mathbf{B}}_k\mathbf{L}\\ \mathbf{K}\widetilde{\mathbf{C}}_k & \widehat{\mathbf{A}} - \mathbf{K}\widetilde{\mathbf{C}}_k - \widehat{\mathbf{B}}\mathbf{L}\end{bmatrix} , \quad \underline{\boldsymbol{\mathcal{W}}}_k = \begin{bmatrix}\underline{\widetilde{\boldsymbol{w}}}_k\\ \mathbf{K}\underline{\widetilde{\boldsymbol{v}}}_k\end{bmatrix} .$$

Having obtained the closed-loop formulation of the considered system, we restate and solve the problem of minimizing (3) in the next section.

C. Optimization Problem and Solution

In the following, we give the results of this paper in Theorems 1 and 2. These results are obtained by restating the problem of minimizing (3) in terms of the second moment of the closed-loop system and applying the *Matrix Minimum Principle* [20].

Theorem 1 *The problem of minimizing (3) can be restated as*

$$\begin{aligned}\min_{\mathbf{P}_k,\mathbf{L},\mathbf{K}} \quad & \textit{trace}\left[\mathrm{E}\left\{\begin{bmatrix}\widetilde{\mathbf{Q}}_k & \mathbf{0}\\ \mathbf{0} & \mathbf{L}^\top\widetilde{\mathbf{R}}_k\mathbf{L}\end{bmatrix}\middle|\,\mathcal{I}_k\right\}\mathbf{P}_k\right]\\ \textit{s.t.} \quad & \mathbf{P}_k = \mathrm{E}\left\{\mathcal{A}_k\mathbf{P}_k\mathcal{A}_k^\top\middle|\,\mathcal{I}_k\right\} + \mathrm{E}\left\{\underline{\boldsymbol{\mathcal{W}}}_k\underline{\boldsymbol{\mathcal{W}}}_k^\top\middle|\,\mathcal{I}_k\right\}\\ & \mathbf{P}_k \geq 0\ .\end{aligned} \tag{7}$$

Proof: The proof follows the argumentation in [21]. Consider the second moment of the closed-loop system

$$\mathbf{P}_k = \mathrm{E}\left\{\left[\begin{bmatrix}\underline{\boldsymbol{\xi}}_k\\ \widehat{\underline{\boldsymbol{\xi}}}_k\end{bmatrix}\begin{bmatrix}\underline{\boldsymbol{\xi}}_k\\ \widehat{\underline{\boldsymbol{\xi}}}_k\end{bmatrix}^\top\right]\middle|\,\mathcal{I}_k\right\} = \begin{bmatrix}\mathbf{P}_k^{11} & \mathbf{P}_k^{12}\\ \mathbf{P}_k^{12\top} & \mathbf{P}_k^{22}\end{bmatrix}$$

and its evolution

$$\begin{aligned}\mathbf{P}_{k+1} &= \mathrm{E}\left\{\begin{bmatrix}\underline{\boldsymbol{\xi}}_{k+1}\\ \widehat{\underline{\boldsymbol{\xi}}}_{k+1}\end{bmatrix}\begin{bmatrix}\underline{\boldsymbol{\xi}}_{k+1}\\ \widehat{\underline{\boldsymbol{\xi}}}_{k+1}\end{bmatrix}^\top\middle|\,\mathcal{I}_k\right\}\\ &= \mathrm{E}\left\{\mathcal{A}_k\mathbf{P}_k\mathcal{A}_k^\top\middle|\,\mathcal{I}_k\right\} + \mathrm{E}\left\{\underline{\boldsymbol{\mathcal{W}}}_k\underline{\boldsymbol{\mathcal{W}}}_k^\top\middle|\,\mathcal{I}_k\right\}\ ,\end{aligned}$$

where we already took the expectation w.r.t. $\underline{\boldsymbol{\xi}}_k$ exploiting the fact that the jump variables $\boldsymbol{\theta}_k$ and $\gamma_{k-i|k}, i \in \{0, 1, \ldots, M-1\}$ are independent of $\underline{\boldsymbol{x}}_k$ and thus also of $\underline{\boldsymbol{\xi}}_k$.

Now, consider the stage costs

$$C_k = \mathrm{E}\left\{\underline{\boldsymbol{x}}_k^\top\mathbf{Q}\underline{\boldsymbol{x}}_k + \underline{\boldsymbol{u}}_k^\top\mathbf{R}\underline{u}_k\middle|\,\mathcal{I}_k\right\}\ .$$

Expressing C_k in terms of the augmented closed-loop state yields

$$\begin{aligned}C_k &= \mathrm{E}\left\{\begin{bmatrix}\underline{\boldsymbol{\xi}}_k\\ \widehat{\underline{\boldsymbol{\xi}}}_k\end{bmatrix}^\top\begin{bmatrix}\widetilde{\mathbf{Q}}_k & \mathbf{0}\\ \mathbf{0} & \mathbf{L}^\top\widetilde{\mathbf{R}}_k\mathbf{L}\end{bmatrix}\begin{bmatrix}\underline{\boldsymbol{\xi}}_k\\ \widehat{\underline{\boldsymbol{\xi}}}_k\end{bmatrix}\middle|\,\mathcal{I}_k\right\}\\ &= \mathrm{trace}\left[\mathrm{E}\left\{\begin{bmatrix}\widetilde{\mathbf{Q}}_k & \mathbf{0}\\ \mathbf{0} & \mathbf{L}^\top\widetilde{\mathbf{R}}_k\mathbf{L}\end{bmatrix}\middle|\,\mathcal{I}_k\right\}\mathbf{P}_k\right]\ ,\end{aligned}$$

with

$$\widetilde{\mathbf{Q}}_k = \begin{bmatrix}\mathbf{Q} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{H}_k^\top\mathbf{R}\mathbf{H}_k\end{bmatrix}\ ,\quad \widetilde{\mathbf{R}}_k = \mathbf{J}_k^\top\mathbf{R}\mathbf{J}_k\ .$$

If $\mathbf{P}_k$ converges to a finite positive semidefinite $\mathbf{P}_\infty$ for $k \to \infty$, the costs C_k converge to C_∞ [21] and we obtain the statement of Theorem 1. ■

Theorem 2 *The solution to the optimization problem (7) is given by the following set of coupled equations:*

$$\begin{aligned}\overline{\boldsymbol{\Psi}} &= \mathrm{E}\left\{\left(\widetilde{\mathbf{A}}_k - \mathbf{K}\widetilde{\mathbf{C}}_k\right)\overline{\boldsymbol{\Psi}}\left(\widetilde{\mathbf{A}}_k - \mathbf{K}\widetilde{\mathbf{C}}_k\right)^\top\right\}\\ &+ \mathrm{E}\left\{\left(\widetilde{\mathbf{A}}_k - \widetilde{\mathbf{B}}_k\mathbf{L}\right)\underline{\boldsymbol{\Psi}}\left(\widetilde{\mathbf{A}}_k - \widetilde{\mathbf{B}}_k\mathbf{L}\right)^\top\right\}\\ &- \left(\widehat{\mathbf{A}} - \widehat{\mathbf{B}}\mathbf{L}\right)\underline{\boldsymbol{\Psi}}\left(\widehat{\mathbf{A}} - \widehat{\mathbf{B}}\mathbf{L}\right)^\top + \widetilde{\mathbf{W}} + \mathbf{K}\widetilde{\mathbf{V}}\mathbf{K}^\top\ ,\end{aligned} \tag{8}$$

$$\begin{aligned}\underline{\boldsymbol{\Psi}} &= \mathrm{E}\left\{\mathbf{K}\widetilde{\mathbf{C}}_k\overline{\boldsymbol{\Psi}}\widetilde{\mathbf{C}}_k^\top\mathbf{K}^\top\right\}\\ &+ \left(\widehat{\mathbf{A}} - \widehat{\mathbf{B}}\mathbf{L}\right)\underline{\boldsymbol{\Psi}}\left(\widehat{\mathbf{A}} - \widehat{\mathbf{B}}\mathbf{L}\right)^\top + \mathbf{K}\widetilde{\mathbf{V}}\mathbf{K}^\top\ ,\end{aligned} \tag{9}$$

$$\begin{aligned}\overline{\boldsymbol{\Lambda}} &= \mathrm{E}\left\{\left(\widetilde{\mathbf{A}}_k - \widetilde{\mathbf{B}}_k\mathbf{L}\right)^\top\overline{\boldsymbol{\Lambda}}\left(\widetilde{\mathbf{A}}_k - \widetilde{\mathbf{B}}_k\mathbf{L}\right)\right\}\\ &+ \mathrm{E}\left\{\left(\widetilde{\mathbf{A}}_k - \widetilde{\mathbf{B}}_k\mathbf{L}\right)^\top\underline{\boldsymbol{\Lambda}}\left(\widetilde{\mathbf{A}}_k - \widetilde{\mathbf{B}}_k\mathbf{L}\right)\right\}\\ &- \left(\widehat{\mathbf{A}} - \widehat{\mathbf{B}}\mathbf{L}\right)^\top\underline{\boldsymbol{\Lambda}}\left(\widehat{\mathbf{A}} - \widehat{\mathbf{B}}\mathbf{L}\right) + \mathbf{L}^\top\widehat{\mathbf{R}}\mathbf{L} + \widehat{\mathbf{Q}}\ ,\end{aligned} \tag{10}$$

$$\begin{aligned}\underline{\boldsymbol{\Lambda}} &= \mathrm{E}\left\{\mathbf{L}^\top\widetilde{\mathbf{B}}_k^\top\overline{\boldsymbol{\Lambda}}\widetilde{\mathbf{B}}_k\mathbf{L}\right\} + \mathbf{L}^\top\widehat{\mathbf{R}}\mathbf{L}\\ &+ \mathrm{E}\left\{\left(\widehat{\mathbf{A}} - \mathbf{K}\widetilde{\mathbf{C}}_k\right)^\top\underline{\boldsymbol{\Lambda}}\left(\widehat{\mathbf{A}} - \mathbf{K}\widetilde{\mathbf{C}}_k\right)\right\}\\ &+ \mathrm{E}\left\{\left(\widetilde{\mathbf{B}}_k\mathbf{L} - \mathbf{K}\widetilde{\mathbf{C}}_k\right)^\top\underline{\boldsymbol{\Lambda}}\left(\widetilde{\mathbf{B}}_k\mathbf{L} - \mathbf{K}\widetilde{\mathbf{C}}_k\right)\right\}\\ &- \mathrm{E}\left\{\left(\widehat{\mathbf{B}}\mathbf{L} - \mathbf{K}\widetilde{\mathbf{C}}_k\right)^\top\underline{\boldsymbol{\Lambda}}\left(\widehat{\mathbf{B}}\mathbf{L} - \mathbf{K}\widetilde{\mathbf{C}}_k\right)\right\}\ ,\end{aligned} \tag{11}$$

$$\begin{aligned}\mathbf{L} &= \left(\mathrm{E}\left\{\widetilde{\mathbf{B}}_k^\top\overline{\boldsymbol{\Lambda}}\widetilde{\mathbf{B}}_k\right\} + \mathrm{E}\left\{\widetilde{\mathbf{B}}_k^\top\underline{\boldsymbol{\Lambda}}\widetilde{\mathbf{B}}_k\right\} - \widehat{\mathbf{B}}^\top\underline{\boldsymbol{\Lambda}}\widehat{\mathbf{B}} + \widehat{\mathbf{R}}\right)^\dagger\\ &\times\left(\mathrm{E}\left\{\widetilde{\mathbf{B}}_k^\top\overline{\boldsymbol{\Lambda}}\widetilde{\mathbf{A}}_k\right\} + \mathrm{E}\left\{\widetilde{\mathbf{B}}_k^\top\underline{\boldsymbol{\Lambda}}\widetilde{\mathbf{A}}_k\right\} - \widehat{\mathbf{B}}^\top\underline{\boldsymbol{\Lambda}}\widehat{\mathbf{A}}\right)\ ,\end{aligned} \tag{12}$$

and

$$\mathbf{K} = \mathrm{E}\left\{\widetilde{\mathbf{A}}_k\overline{\boldsymbol{\Psi}}\widetilde{\mathbf{C}}_k\right\}\left(\mathrm{E}\left\{\widetilde{\mathbf{C}}_k\overline{\boldsymbol{\Psi}}\widetilde{\mathbf{C}}_k^\top\right\} + \widetilde{\mathbf{V}}\right)^\dagger\ , \tag{13}$$

where

$$\begin{aligned}\widehat{\mathbf{Q}} &= \mathrm{E}\left\{\widetilde{\mathbf{Q}}_k\middle|\,\mathcal{I}_k\right\}\ ,\quad \widehat{\mathbf{R}} = \mathrm{E}\left\{\widetilde{\mathbf{R}}_k\middle|\,\mathcal{I}_k\right\}\ ,\\ \widetilde{\mathbf{W}} &= \mathrm{E}\left\{\underline{\widetilde{\boldsymbol{w}}}_k\underline{\widetilde{\boldsymbol{w}}}_k^\top\middle|\,\mathcal{I}_k\right\}\ ,\quad \widetilde{\mathbf{V}} = \mathrm{E}\left\{\underline{\widetilde{\boldsymbol{v}}}_k\underline{\widetilde{\boldsymbol{v}}}_k^\top\middle|\,\mathcal{I}_k\right\}\ .\end{aligned}$$

In equations (8)-(13), the expectation is taken w.r.t. the mode $\boldsymbol{\theta}_k$ and the measurement availability $\boldsymbol{\gamma}_k$. For the former, the controller assumes the limit distribution $\underline{\widehat{\theta}}$, and for the latter the individual measurement availability probabilities that can be calculated from $f^{SC}(\boldsymbol{\tau}^{SC})$.

Proof: Introducing the Lagrange multiplier

$$\boldsymbol{\Lambda} = \begin{bmatrix}\boldsymbol{\Lambda}_{11} & \boldsymbol{\Lambda}_{12}\\ \boldsymbol{\Lambda}_{12}^\top & \boldsymbol{\Lambda}_{22}\end{bmatrix}\ ,$$

the Hamiltonian of (7) is given by

$$\begin{aligned}\mathcal{L}(\mathbf{P}_k, \mathbf{L}, \mathbf{K}, \boldsymbol{\Lambda}) &= \mathrm{trace}\left[\mathrm{E}\left\{\begin{bmatrix}\widetilde{\mathbf{Q}}_k & \mathbf{0}\\ \mathbf{0} & \mathbf{L}^\top\widetilde{\mathbf{R}}_k\mathbf{L}\end{bmatrix}\middle|\,\mathcal{I}_k\right\}\mathbf{P}_k\right]\\ &+ \mathrm{trace}\left[\boldsymbol{\Lambda}\Big(\mathrm{E}\left\{\mathcal{A}_k\mathbf{P}_k\mathcal{A}_k^\top\middle|\,\mathcal{I}_k\right\} + \mathrm{E}\left\{\underline{\boldsymbol{\mathcal{W}}}_k\underline{\boldsymbol{\mathcal{W}}}_k^\top\middle|\,\mathcal{I}_k\right\} - \mathbf{P}_k\Big)\right]\ ,\end{aligned}$$

and the optimization problem becomes

$$\begin{aligned}\min_{\mathbf{P}_k,\mathbf{L},\mathbf{K},\boldsymbol{\Lambda}} \quad & \mathcal{L}(\mathbf{P}_k, \mathbf{L}, \mathbf{K}, \boldsymbol{\Lambda})\\ \text{s.t.} \quad & \mathbf{P}_k \geq 0\ ,\ \boldsymbol{\Lambda} \geq 0\ .\end{aligned} \tag{14}$$

Necessary conditions for the minimum of (14) are [20]

$$\frac{\partial\mathcal{L}}{\partial\mathbf{P}_k} = \mathbf{0},\ \frac{\partial\mathcal{L}}{\partial\mathbf{L}} = \mathbf{0},\ \frac{\partial\mathcal{L}}{\partial\mathbf{K}} = \mathbf{0},\ \frac{\partial\mathcal{L}}{\partial\boldsymbol{\Lambda}} = \mathbf{0}\ .$$

Differentiation w.r.t. $\mathbf{P}_k$ and $\boldsymbol{\Lambda}$ yields

$$\mathbf{P}_k = \mathrm{E}\left\{\mathcal{A}_k\mathbf{P}_k\mathcal{A}_k^\top\middle|\,\mathcal{I}_k\right\} + \mathrm{E}\left\{\underline{\boldsymbol{\mathcal{W}}}_k\underline{\boldsymbol{\mathcal{W}}}_k^\top\middle|\,\mathcal{I}_k\right\} \tag{15}$$

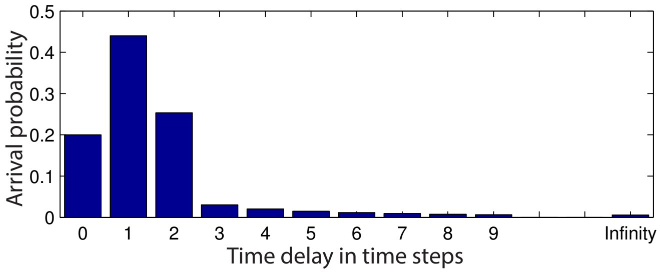

Fig. 2. Delay probability function used for the CA- and the SC-link in simulations.

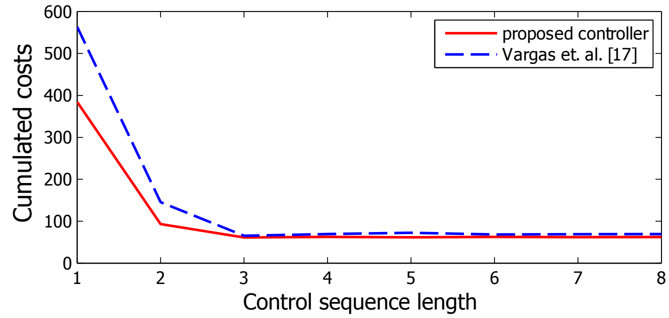

Fig. 3. Cumulated costs of the proposed controller and the control law presented in [18].

and

$$\mathbf{\Lambda} = \mathrm{E}\left\{ \begin{bmatrix} \widetilde{\mathbf{Q}}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{L}^\top \widetilde{\mathbf{R}}_k \mathbf{L} \end{bmatrix} \middle| \mathcal{I}_k \right\} + \mathrm{E}\left\{ \mathcal{A}_k^\top \mathbf{\Lambda} \mathcal{A}_k \middle| \mathcal{I}_k \right\} . \tag{16}$$

At this point, we define

$$\overline{\mathbf{\Psi}} = \mathbf{P}_k^{11} - \mathbf{P}_k^{22} \quad \text{and} \quad \underline{\mathbf{\Psi}} = \mathbf{P}_k^{22} \tag{17}$$

$$\overline{\mathbf{\Lambda}} = \mathbf{\Lambda}_{11} - \mathbf{\Lambda}_{22} \quad \text{and} \quad \underline{\mathbf{\Lambda}} = \mathbf{\Lambda}_{22} \ . \tag{18}$$

According to [21], the optimality implies

$$\mathbf{P}_{12} = \underline{\mathbf{\Psi}} \quad \text{and} \quad \mathbf{\Lambda}_{12} = -\underline{\mathbf{\Lambda}}$$

and according to [22]

$$\underline{\mathbf{\Psi}} > 0 \quad \text{and} \quad \underline{\mathbf{\Lambda}} > 0 \ .$$

Finally, equations (8-11) are obtained after a lengthy straightforward manipulation of (15) and (16) using definitions (17) and (18). Equations (12) and (13) can be obtained by differentiating (14) w.r.t. $\mathbf{L}$ and $\mathbf{K}$ using the definitions (17) and (18), and applying differentiation rules from [23] and [24]. ■

An analytic solution to the set of coupled equations in Theorem 2 is hard to obtain [21]. Thus, we propose to solve these equation applying an iterative scheme. This scheme is given in the appendix.

The equations given in Theorem 2 provide only the necessary conditions for the optimum. In the future work, it is necessary to investigate if these conditions are also sufficient.

V. NUMERICAL EXAMPLE

In order to evaluate the presented control law, we compared it with the control law presented in [18] by means of simulation. Because the control law presented by Vargas et. al. [18] is based on state feedback, we used the filter presented in [25] to provide it with a state estimate. The simulated system was chosen to

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{W} = \begin{bmatrix} 0.4^2 & 0 \\ 0 & 0.4^2 \end{bmatrix}, \mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$\mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \mathbf{V} = 0.8^2, \mathbf{R} = 1,$$

with the initial state

$$\underline{\overline{x}}_0 = \begin{bmatrix} 100 \\ 0 \end{bmatrix}, \ \mathbf{X}_0 = \begin{bmatrix} 0.5^2 & 0 \\ 0 & 0.5^2 \end{bmatrix} \ .$$

For both networks, the CA- and the SC- link, we used the network whose delay probability distribution is depicted in Fig. 2.

With this setup, we conducted 6000 Monte-Carlo runs with $K = 100$ time steps each for different buffer lengths M and different sequence lengths N. Fig. 3 depicts the average cumulated costs

$$J = \frac{1}{K} \sum_{k=0}^{K} \left(\underline{\boldsymbol{x}}_k^\top \mathbf{Q} \underline{\boldsymbol{x}}_k + \underline{u}_k^\top \mathbf{R} \underline{u}_k \right)$$

over control sequence length induced by both control laws. The length of the measurement buffer was set to $M = 8$. It can be seen that the costs of both control laws converge to a fixed value for control sequence lengths $N \geq 3$. Although our proposed control law is time-invariant, it induces slightly lower costs than the combination of the control law presented by Vargas et. al. and the filter from [25]. This is due to the fact that for the combination of the control law [18] and the filter [25], we assumed the separation between control and estimation. However, our control law respects the fact that the separation principle does not hold.

The implementation of the proposed control law is available at the CloudRunner project website [26].

VI. CONCLUSION

In this paper, we considered sequence-based control over networks that do not provide acknowledgments and that are subject to packet losses and delays. In this setup, the separation principle between control and estimation does not hold and thus, the optimal control law is nonlinear. However, with the structural assumption that the estimator and the controller gains are constant, we derived the optimal linear infinite-horizon control law. Even though the control law is linear, we need to solve a nonlinear constrained optimization problem in order to obtain it. We give the solution to this optimization problem as a set of coupled equations.

Our future research will concentrate on derivation of stability criteria based on control sequence length, the measurement buffer size, and the network characteristics. Further, we will investigate properties of the local minima concerning optimality and the convergence of the iterative solution if the coupled equations to these points. Another important topic is the sufficiency of the optimality conditions. Finally, an extensive evaluation of the proposed approach with regard to parameters (control sequence length, measurement buffer length) will be provided.

APPENDIX

A. Iterative Control Law Computation

Equations (8-13) can be solved iteratively according to

$$
\begin{aligned}
\overline{\mathbf{\Psi}}_{k+1} &= \mathrm{E}\left\{\left(\widetilde{\mathbf{A}}_k - \mathbf{K}_k\widetilde{\mathbf{C}}_k\right)\overline{\mathbf{\Psi}}_k\left(\widetilde{\mathbf{A}}_k - \mathbf{K}_k\widetilde{\mathbf{C}}_k\right)^\top\right\}\\
&\quad + \mathrm{E}\left\{\left(\widetilde{\mathbf{A}}_k - \widetilde{\mathbf{B}}_k\mathbf{L}_k\right)\underline{\mathbf{\Psi}}_k\left(\widetilde{\mathbf{A}}_k - \widetilde{\mathbf{B}}_k\mathbf{L}_k\right)^\top\right\}\\
&\quad - \left(\widehat{\mathbf{A}} - \widehat{\mathbf{B}}\mathbf{L}_k\right)\underline{\mathbf{\Psi}}_k\left(\widehat{\mathbf{A}} - \widehat{\mathbf{B}}\mathbf{L}_k\right)^\top + \widetilde{\mathbf{W}} + \mathbf{K}_k\widetilde{\mathbf{V}}\mathbf{K}_k^\top\ ,\\
\underline{\mathbf{\Psi}}_{k+1} &= \mathrm{E}\left\{\mathbf{K}_k\widetilde{\mathbf{C}}_k\overline{\mathbf{\Psi}}_k\widetilde{\mathbf{C}}_k^\top\mathbf{K}_k^\top\right\}\\
&\quad + \left(\widehat{\mathbf{A}} - \widehat{\mathbf{B}}\mathbf{L}_k\right)\underline{\mathbf{\Psi}}_k\left(\widehat{\mathbf{A}} - \widehat{\mathbf{B}}\mathbf{L}_k\right)^\top + \mathbf{K}_k\widetilde{\mathbf{V}}\mathbf{K}_k^\top\ ,\\
\overline{\mathbf{\Lambda}}_{k+1} &= \mathrm{E}\left\{\left(\widetilde{\mathbf{A}}_k - \widetilde{\mathbf{B}}_k\mathbf{L}_k\right)^\top\overline{\mathbf{\Lambda}}_k\left(\widetilde{\mathbf{A}}_k - \widetilde{\mathbf{B}}_k\mathbf{L}_k\right)\right\}\\
&\quad + \mathrm{E}\left\{\left(\widetilde{\mathbf{A}}_k - \widetilde{\mathbf{B}}_k\mathbf{L}_k\right)^\top\underline{\mathbf{\Lambda}}_k\left(\widetilde{\mathbf{A}}_k - \widetilde{\mathbf{B}}_k\mathbf{L}_k\right)\right\}\\
&\quad - \left(\widehat{\mathbf{A}} - \widehat{\mathbf{B}}\mathbf{L}_k\right)^\top\underline{\mathbf{\Lambda}}_k\left(\widehat{\mathbf{A}} - \widehat{\mathbf{B}}\mathbf{L}_k\right) + \mathbf{L}_k^\top\widehat{\mathbf{R}}\mathbf{L}_k + \widehat{\mathbf{Q}}\ ,\\
\underline{\mathbf{\Lambda}}_{k+1} &= \mathrm{E}\left\{\mathbf{L}_k^\top\widetilde{\mathbf{B}}_k^\top\overline{\mathbf{\Lambda}}_k\widetilde{\mathbf{B}}_k\mathbf{L}_k\right\} + \mathbf{L}_k^\top\widehat{\mathbf{R}}\mathbf{L}_k\\
&\quad + \mathrm{E}\left\{\left(\widehat{\mathbf{A}} - \mathbf{K}_k\widetilde{\mathbf{C}}_k\right)^\top\underline{\mathbf{\Lambda}}_k\left(\widehat{\mathbf{A}} - \mathbf{K}_k\widetilde{\mathbf{C}}_k\right)\right\}\\
&\quad + \mathrm{E}\left\{\left(\widetilde{\mathbf{B}}_k\mathbf{L}_k - \mathbf{K}_k\widetilde{\mathbf{C}}_k\right)^\top\underline{\mathbf{\Lambda}}_k\left(\widetilde{\mathbf{B}}_k\mathbf{L}_k - \mathbf{K}_k\widetilde{\mathbf{C}}_k\right)\right\}\\
&\quad - \mathrm{E}\left\{\left(\widehat{\mathbf{B}}\mathbf{L}_k - \mathbf{K}_k\widetilde{\mathbf{C}}_k\right)^\top\underline{\mathbf{\Lambda}}_k\left(\widehat{\mathbf{B}}\mathbf{L}_k - \mathbf{K}_k\widetilde{\mathbf{C}}_k\right)\right\}\ ,\\
\mathbf{L}_{k+1} &= \left(\mathrm{E}\left\{\widetilde{\mathbf{B}}_k^\top\overline{\mathbf{\Lambda}}_k\widetilde{\mathbf{B}}_k\right\} + \mathrm{E}\left\{\widetilde{\mathbf{B}}_k^\top\underline{\mathbf{\Lambda}}_k\widetilde{\mathbf{B}}_k\right\} - \widehat{\mathbf{B}}^\top\underline{\mathbf{\Lambda}}_k\widehat{\mathbf{B}} + \widehat{\mathbf{R}}\right)^\dagger\\
&\quad \times\left(\mathrm{E}\left\{\widetilde{\mathbf{B}}_k^\top\overline{\mathbf{\Lambda}}_k\widetilde{\mathbf{A}}_k\right\} + \mathrm{E}\left\{\widetilde{\mathbf{B}}_k^\top\underline{\mathbf{\Lambda}}_k\widetilde{\mathbf{A}}_k\right\} - \widehat{\mathbf{B}}^\top\underline{\mathbf{\Lambda}}_k\widehat{\mathbf{A}}\right)\ ,\\
\mathbf{K}_{k+1} &= \mathrm{E}\left\{\widetilde{\mathbf{A}}_k\overline{\mathbf{\Psi}}_k\widetilde{\mathbf{C}}_k\right\}\left(\mathrm{E}\left\{\widetilde{\mathbf{C}}_k\overline{\mathbf{\Psi}}_k\widetilde{\mathbf{C}}_k^\top\right\} + \widetilde{\mathbf{V}}\right)^\dagger\ ,
\end{aligned}
$$

with initial conditions

$$
\overline{\mathbf{\Psi}}_0 = \mathbf{0},\ \underline{\mathbf{\Psi}}_0 = \mathbf{0},\ \overline{\mathbf{\Lambda}}_0 = \mathbf{0},\ \ \underline{\mathbf{\Lambda}}_0 = \mathbf{0},\ \mathbf{L}_0 = \mathbf{0},\ \mathbf{K}_0 = \mathbf{0}\ .
$$

ACKNOWLEDGMENTS

The authors want to thank Alessandro Vargas for providing the implementation of the control law he presented in [18]. This work was supported by the German Science Foundation (DFG) within the Research Training Group RTG 1194 “Self-organizing Sensor-Actuator-Networks”.

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